# MODELLING THE PENSION SYSTEM IN AN OVERLAPPING-GENERATIONS GENERAL EQUILIBRIUM MODELLING FRAMEWORK[1]

**MIROSLAV VERBIČ***

ABSTRACT: *This article presents a theoretical contribution to the field of overlapping-generations general equilibrium modelling, i.e. an upgrade of this branch of models with a pension system. Within the pension block we model both the first pension pillar, financed on a pay-as-you-go basis, and the fully-funded second pillar of the Slovenian pension system. The modelling of the first pension pillar is based on cash flows of the mandatory pension insurance institution, the relationship between the pension base and the pension, and the process of harmonising pension growth with wage growth. The modelling of the second pillar centres on implementation of the liquidity constraint. Use was made of supplementary pension profiles, and the ratio between premia paid and pensions paid out from supplementary pension insurance. The total pension category was also introduced and the model ensured that at every point households adjusted the scope of labour supply and their current consumption towards the target total pension.*

**Keywords:** *First pension pillar; General equilibrium models; Liquidity constraint; MCP; Overlapping generations; PAYG; Pension system; Second pension pillar*

**UDC:** 368.914(497.4)

**JEL Classification:** C68; D58; D91; H55

## 1. INTRODUCTION

Effectively monitoring the consequences of economic policy on social development demands an appropriate tool, namely one capable of reflecting the complex consequences of the impact of overall and individual social and tax policy measures for households and the national budget. Overlapping-generations general equilibrium (OLG-GE) models currently represent the most advanced form of numerical general equilibrium models

[1] This research was funded jointly by the Ministry of Finance, Ministry of the Economy, Ministry of Health, and Ministry of Labour, Family and Social Affairs of the Republic of Slovenia under contract nos. 3311-02-828610 and 3311-04-828915. I am very grateful for the data and very helpful comments from Renger van Nieuwkoop, Boris Majcen, Tine Stanovnik, Jože Sambt, Mateja Slapar, and two anonymous referees. Responsibility for the views expressed and any remaining errors is mine alone.

* Institute for Economic Research, Kardeljeva ploščad 17, 1109 Ljubljana, Slovenia, E-mail: miroslav.verbic@guest.arnes.si

and it is for this reason we find them suitable for this task. The case in hand involves a dynamic model of a national economy including overlapping generations of various households, distinguished according to size of household and income level, which maximise the total utility over the lifespan, assuming perfect foresight. This kind of model facilitates the monitoring and forecasting of complex short-term and long-term consequences of demographic changes – (continued) ageing of the population being key among them – on individual categories of public finance, as well as the impact of changes in the tax system and social security system on the flexibility, competitiveness and thus growth of the economy.

The advantages that overlapping-generations equilibrium models offer in comparison to other modelling tools, such as actuarial models of pension reforms and generational accounting models, are not to be searched for directly in the modelling of specific socio-economic phenomena such as a demographic slowdown of GDP, but first and foremost in the key characteristics of general equilibrium modelling which are closer to the functioning of the actual economy, making the results of the model more realistic and reliable. Of course, this entails modelling mutual interactions and feedback effects between macro-economic aggregates that simpler models are unable to capture. This is seen in the analysis of the pension system, where a link has to be established between labour endowment and labour price; unfavourable demographic changes will lead to a reduction in the active working population and hence the labour endowment, which will lead to an increase in the price of labour (wages) above steady state growth. Since pension dynamics depend on the dynamics of wages, this also means higher pension expenditure. It can be seen that modelling relationships of this kind is vital for ensuring a realistic and accurate analysis is produced using a model of this kind.

The contribution of this article to overlapping-generations general equilibrium modelling relates to modelling the pension system within a dynamic general equilibrium framework, both the mandatory pay-as-you-go (PAYG) financed component as well as the fully-funded (FF) supplementary component of pension insurance. Studying the modelling of the pension system should be addressed primarily within the context of implementing the transition from a pay-as-you-go financed system to a fully-funded system, and other ongoing changes in pension legislation that are becoming increasingly prominent in Slovenia. The second chapter thus provides a brief description of the SIOLG 2.0 dynamic general equilibrium model of the Slovenian economy, which will serve as the basis for modelling the pension system. The third chapter covers the modelling of the first pillar of the pension block, considering the mandatory pension insurance institution, the relationship between the pension base and the pension and addressing the relationship between premia paid and pensions paid out from supplementary insurance, which will finally enable us to model supplementary pension insurance within the pension block of our model (the fourth chapter). The fifth chapter concludes the article with the key findings.

## 2. THE OLG-GE MODEL OF THE SLOVENIAN ECONOMY

The model SIOLG 2.0 is a dynamic overlapping-generations general equilibrium model of the Slovenian economy[2], based on social accounting matrix (SAM) for the year 2000, data on the demographic structure of the population, expected future demographic developments, characteristics of Slovenian households, and the breakdown of households into generations (Verbič 2007). The model has been developed with the very intention of analysing the sustainability of Slovenian public finances[3], although it can be used to analyse any part or any sector of the economy.

The starting points of the OLG-GE model are the life cycle theory of consumption by Modigliani and Brumberg (1954) and the permanent income hypothesis by Friedman (1957), which are actually special cases of the more general theory of the intertemporal allocation of consumption (Deaton 1992). Unlike in Keynes' theory of behaviour of consumption and savings, based only on current income, in the OLG-GE model consumption and savings are derived from intertemporal optimisation behaviour and are therefore dependent on full lifetime income. In the simplest case of unchanged income until retirement (Modigliani 1986), consumers save during their active lifetime and spend their savings after the retirement in order to maintain unchanged consumption. The retirement is therefore the *raison d'etre* for saving.

Overlapping-generations general equilibrium models represent the pinnacle of dynamic CGE modelling. OLG-GE modelling was first proposed already by Samuelson (1958) and Diamond (1965), but did not become an established means of economic modelling until Auerbach and Kotlikoff (1987) who constructed a relatively large and detailed computable model of the American economy based on a detailed decomposition of the consumption side of the model which means that, unlike in the Ramsey-type models, consumers live a finite length of time but long enough to live at least one period with the next generation of consumers. Defining consumers by their birth cohort enables an analysis of inter-generational effects which makes OLG-GE models especially valuable for the analysis of tax policies, pension policies and other social policies.

Besides the path-breaking work of Auerbach and Kotlikoff (1987), one should emphasise a number of contributions that have significantly influenced the field of OLG-GE modelling. Rasmussen and Rutherford (2004) provided a compact approach to the numerical simulation of overlapping-generations models with perfect foresight and finite lifetimes. Their exposition presented the fundamental characteristics of this class of problems, formulated using the complementarity framework. They addressed some important issues regarding the representation of international trade, bequests, government tax and ex-

[2] The equations of the model SIOLG 2.0 (excluding the pension system, the representation of which is the main purpose of this article) are presented in Appendix 1. The variables of the model (including the pension system) are presented in Appendix 2.

[3] Readers are invited to consult Verbič *et al.* (2006), Verbič (2007), and Verbič (2007a) in order to obtain an insight into the consequences of the ageing population in Slovenia for welfare, macroeconomic aggregates, supplementary pension saving and public finance sustainability.

penditure policies, and the labour-leisure choice with endogenous retirement. Hviding and Mérette (1998) and Fougère and Mérette (2000) incorporated endogenous growth, which was generated by the accumulation of both physical and human capital. They established that the estimates of the long-run economic effects of population ageing are significantly altered when the model features endogenous growth. Equipe Ingénue (2001; 2001a) developed a multiregional world model in which the structure of each regional economy is similar to that of other OLG general equilibrium models, except for the exogenous labour supply. Aglietta *et al.* (2007) and Equipe Ingénue (2007) upgraded the initial model with imperfect financial markets, stochastic life expectancy and bequest motives.

The dynamic general equilibrium model SIOLG 2.0 comprises not only the standard model structure of a national economy, but also the demographic block and the pension block, within the framework of which the first and second pillar of the Slovenian pension system are being modelled. Since the model incorporates most of the contemporary techniques of CGE modelling, the extent to which this field in Slovenia lagged behind the rest of the world has practically been eliminated. Namely, the model is built within the general algebraic modelling system (GAMS) which has become both the most widely used programming language and most widespread computer software (Brooke *et al.* 1998) for constructing and solving large and complex CGE models.

Within the GAMS framework, the dynamic general equilibrium model is written in Mathiesen's (1985) formulation of the Arrow-Debreu (1954) equilibrium model, i.e. as a mixed-complementarity problem (MCP). The key advantage of this formulation is the compact presentation of the general equilibrium problem which is achieved by treating variables implicitly and thus significantly reducing the computation time for higher-dimensional models. Namely, the mathematical programme includes equalities as well as inequalities, where the complementarity slackness holds between system variables and system conditions (Rutherford 1995a; Böhringer *et al.* 2003). Functions of the model are written in Rutherford's (1995) calibrated share form; a reasonably straightforward algebraic transformation which nevertheless considerably simplifies the calibration of the model (Böhringer *et al.* 2003; Balistreri and Hillberry 2003). To solve the model, i.e. to achieve convergence, a recent version of the PATH solver (Ferris and Munson 2000) is used, which is renowned for its computational efficiency.

Consumers live in the model according to their expected length of life, i.e. their life expectancy at birth. Assuming that the life expectancy is approximately 80 years and that the active lifetime period starts at the age of 20, there are 60 generations in each period of the model. There is a new cohort of consumers born in each such period, thus increasing the population while at the same time a number of consumers die and decrease the total population. Consumers are observed in five-year intervals within households which maximise the expected lifetime utility subject to their income constraints, where one has to put out the need to save for retirement and to support children. Households are differentiated in the model according to year of birth, income and size; within each cohort a distinction is made between a couple without children and a nuclear family with two children on average, and five income profiles representing different income brackets. Consequently, there are ten versions of the model altogether, which facilitates an analysis of intra-generational effects of different economic policies.

The volume of labour and labour productivity growth are given exogenously. Changes in wages are reflected in changes in the labour supply. The consumption of households with children is additionally corrected due to the extra cost per child where children are born in the childbearing age of a woman or, to be precise, a household, i.e. in the age bracket of 20-40 years. In the first ten years after retirement the household comprises two adults, and then one adult. Saving decisions of households affect investment decisions of firms in the capital markets and thus future production. The effects ascribed herein have recurrent effects on the product market through decreasing prices and on the labour market through higher productivity, leading to higher wages and finally the higher income of households. Both effects can be analysed with a dynamic OLG-GE model quite straightforwardly.

The perfect foresight assumption in the forward-looking model specification implies the ability of households to perform the intertemporal optimisation of the present value of entire future consumption. In other words, consumers have full information at their disposal, on average adopt the right decisions and are familiar with future modifications of key economic indicators, which is the quintessence of rational expectations. They are able to anticipate new policies and prepare themselves for future changes. The assumption of an equilibrium in all markets and the assumption of achieved sustainable economic growth enable the analysis of different scenarios which cause deviations from the reference growth path and changes in macroeconomic and microeconomic indicators. This is especially important when analysing social security because it enables a projection of the effects of demographic changes on the social security system. For this we have available three variants of demographic projections; the low variant combines lower fertility with lower life expectancy and lower net migration, while the high variant combines higher fertility with higher life expectancy and higher net migration than in the reference medium variant.

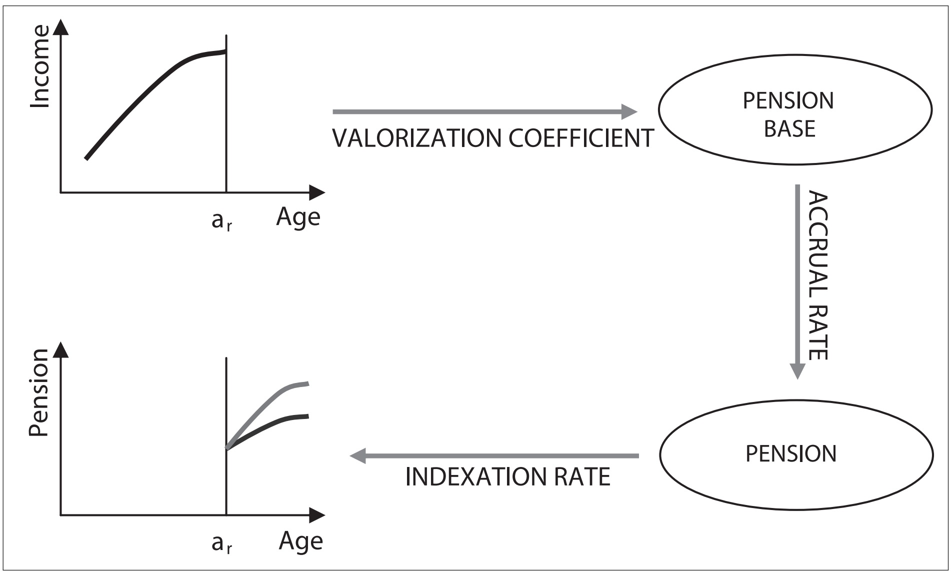
On the other hand, the assumption of perfect foresight is also valid for firms which maximise profits within an environment of perfect competition. Technology is given by the constant elasticity of substitution (CES) production function. The number of production sectors in the model depends on the availability of the input-output table for the base year, which means there are 60 sectors of the standard classification of activities (SCA) available for discretionary aggregation. Government spending depends on economic growth and growth of the population, and is financed with revenues from personal income tax, capital income tax, value-added tax and import duties. The sources of revenue for the Slovenian system of public finances represent various possibilities of funding different economic policies in the simulation phase of the modelling.

The dynamic general equilibrium model SIOLG 2.0 is closed using Armington's (1969) assumption of imperfect substitutability whereby commodities are separated by their source into domestic and imported products. Demand for imported products is derived from the cost minimisation criterion of firms and the utility maximisation criterion of consumers. As regards the export side of the model, domestically produced products are sold at home and abroad but are nevertheless treated as imperfect substitutes. Slovenia is assumed to be a small open economy, implying that changes in the volumes of imports and exports do not affect the terms of trade. International capital flows are endogenous given the intertemporal balance of payments constraint.

## 3. MODELLING THE FIRST PENSION PILLAR

Activities within the basic pension system in the Republic of Slovenia can be divided into at least two parts: (1) activities that occur before the pension accrual when the insured person reaches retirement age, $a_r$, and are actually linked to the accrual process; and (2) activities that occur after pension accrual that are linked to changes in the pension value over time. In order to present the otherwise complex mandatory pension system in Slovenia in a more understandable manner, the first part of the mandatory pension insurance activities are further broken down into the valuation of the pension base and pension accrual. This makes three phases of pension activity: (1) valuation of pension base; (2) pension accrual; and (3) harmonising pension with selected national economic indicators (indexation). The first two phases take place simultaneously, while the third phase follows. The mandatory pension insurance system is thus described in Figure 1.

FIGURE 1: *Functioning of the first pension pillar in Slovenia*




Below we describe the structure of the first pillar of the Slovenian pension system as modelled in the SIOLG 2.0 dynamic general equilibrium model of the Slovenian economy. The modelling can roughly be divided into: (1) revenues of the Institute for Pension and Disability Insurance and total pension expenditure; (2) (minimum) pension bases and pensions of households of an individual generation; and (3) pension indexation (harmonising pension growth with the growth of wages).

### 3.1 The Mandatory Pension Insurance Institution

In Slovenia the mandatory pension insurance institution is known as the Institute of Pension and Disability Insurance (IPDI). Since the article deals with five-year intervals, one can *de facto* discuss mandatory pension institution revenues and expenditure at a five-year level in this model. They are modelled in the form of the income of IPDI on one hand, and total pensions expenditure on the other.

The income of the IPDI, $inc_{IPDI,t}$, primarily comprise mandatory pension contributions (contribution rate $t_l^{rent}$) and where required general government transfers, while alternatively mandatory pension insurance may also be financed with revenues from tax on labour income (via replacement contribution rate $\tau_{l,t}^{rent}$, applied to the gross labour cost $(1+t_l)\,p_{el,t,g,h}y_{l,t,g,h}$), tax on capital income (with replacement tax rate $\tau_r$), or value-added tax (with replacement tax rate $\tau_{VAT,t}$). This gives the following expression with optional components to match the model scenarios:

$$
\begin{aligned}
inc_{IPDI,t} = {} & \sum_h \sum_g \left[ \left( t_l^{rent} + \tau_{l,t}^{rent} \right) \left( 1 + t_l \right) p_{el,t,g,h} y_{l,t,g,h} \right] + \\
& + \tau_r p_{r,t} \sum_s \left[ \overline{R}_{0,s} y_{t,s} \left( \frac{p_{va,t,s} (1 + \bar{t}_r)}{p_{r,t} \left( 1 + t_r + \tau_r \right)} \right)^{\sigma_{kl}} \right] + \\
& + \sum_s \left\{ \frac{p_{a,t,s} \tau_{VAT,t}}{\left( 1 + \bar{t}_{VAT,s} \right)} \sum_h \sum_g \left[ \beta_{h,s} \bar{c}_{t,g,h} y_{c,t,g,h} \left( \frac{p_{c,t,g,h} \left( 1 + \bar{t}_{VAT,s} \right)}{p_{a,t,s} \left( 1 + t_{VAT,s} + \tau_{VAT,t} \right)} \right)^{\sigma_{cc}} \right] \right\} + \\
& + \sum_s \left( \tau_{VAT,t} p_{A,t,s} \bar{g}_{0,s} y_{G,t} p_{G,t} \frac{1 + \bar{t}_{VAT,s}}{p_{A,t,s} \left( 1 + t_{VAT,s} + \tau_{VAT,t} \right)} \right) + \bar{p}_t \bar{q}_t p_f \bar{\zeta}_{IPDI,0},
\end{aligned}
\tag{1}
$$

where $\bar{q}_t$ is the reference steady state level, $\bar{p}_t$ the reference steady state price, $\bar{c}_{t,g,h}$ the reference private consumption, $y_{c,t,g,h}$ the level of private consumption, $p_{c,t,g,h}$ the price of private consumption, $\beta_{h,s}$ the share of sector $s$ goods in material consumption, $y_{t,s}$ the level of domestic production, $p_{A,t,s}$ the price of Armington goods, $p_{va,t,s}$ price of value added, $y_{l,t,g,h}$ the level of labour supply, $p_{el,t,g,h}$ the price of leisure, $\overline{R}_{t,s}$ reference capital services, $p_{r,t}$ the price of capital services, $\bar{g}_t$ reference government consumption, $y_{G,t}$ level of government consumption, $p_{G,t}$ price of government consumption, $p_f$ price of foreign currency, $\bar{\zeta}_{IPDI,t}$ reference government transfers to the IPDI, $\sigma_{cc}$ substitution elasticity between consumption components, and $\sigma_{kl}$ substitution elasticity between labour and capital.

The total pension expenditure of IPDI equals $\sum_h \sum_g aggpens_{t,g,h}$, where the aggregate pension, $aggpens_{t,g,h}$, paid to household $h$ of generation $g$, is expressed as follows:

$$aggpens_{t,g,h} = \left[ \mu_B + (1-\mu_B)\, \mu_{t,g}^{aret} \nu_{g,h}^{rh} \theta_{g,h}^{\#} \right] pens_{t,g,h}, \tag{2}$$

where $\mu_B$ is the benchmark scenario multiplier (where it equals 1, otherwise it is 0), $\mu_{t,g}^{aret}$ is the multiplier for time periods in which generation $g$ is retired, $\nu_{g,h}^{rh}$ is the number of retired households, $\theta_{g,h}^{\#}$ the number of adults in a household and $pens_{t,g,h}$ is the pension of each retired household $h$ of generation $g$ in time period $t$.

### 3.2 The Pension Base and the Pension

Individual pensions, $pens_{t,g,h}$, are expressed in the benchmark scenario as:

$$pens_{t,g,h} = \mu_B \overline{pens}_{t,g,h}, \tag{3}$$

while their calculation in counterfactual scenarios is expressed separately for persons who already retired in the first period:

$$pens_{t,g,h} = (1-\mu_B)\, \mu_{t,g,h}^{alive} \xi \iota_{t,g}^{rent} \alpha_g pb_{0,h}^{mm} (1+\gamma)^{-5(12-\mathrm{ord}[g])} \tag{4}$$

and for persons who have not yet retired:

$$pens_{t,g,h} = (1-\mu_B)\, \mu_{t,g,h}^{wh} \xi \iota_{t,g}^{rent} \alpha_g pb_{g,h}^{mm}, \tag{5}$$

where $\overline{pens}_{t,g,h}$ is the benchmark pension, $\mu_{t,g,h}^{alive}$ the multiplier for households already existing in the first model period, $\mu_{t,g,h}^{wh}$ the multiplier for households that live entirely within the model horizon, $\xi$ correction factor for the calculation of pensions, required due to the differences between the model calculations of pensions and the IPDI procedures, $\iota_{t,g}^{ren}$ the pension index for calculating the pension of generation $g$, $\alpha_g$ the accrual rate for calculating the pension of generation $g$, $pb_{g,h}^{mm}$ the pension base for calculating the pension of generation $g$ with a correction for the minimum and maximum pension base, and $\gamma$ is the steady state growth rate.

For those people who already retired in the first model period, there are no income profiles available to calculate their pensions in this period which in turn demanded an alternative solution. Expression (4) therefore relates to the pension base:

$$\frac{pb_{0,h}^{mm}}{(1+\gamma)^{5(12-\mathrm{ord}[g])}} = \frac{pb_{0,h}^{mm}}{(1+\gamma)^{60-5\mathrm{ord}[g]}},$$

where $a_r = 60$ is the model retirement age in the base year, we are dealing with five-year age intervals, and ord($g$) is a mathematical operation that assigns a numerical value to the elements of set $g$, i.e. to the years of birth of individual generations.

Pursuant to the new pension law of 1999, the pension base is calculated on the basis of the best consecutive 18 years of income from employment (gross wage). For the purposes of this OLG-GE model these are assumed to be the last 18 years. Since the dynamic model SIOLG 2.0 is based on five-year intervals, an approximation of the pension base is arrived at using the best 17.5 years instead of 18 years of wages.

Wages for period *t* are not valued using a wage index for the period, but with the pension index also used by Wiese (2004). The pension index was the same as the wage index until 1990, but from then on it started to be adjusted according to a relatively untransparent system described in detail in Verbič (2007: 200-203). The valuation process subsequently had a significant negative impact on the replacement rate, as pension indexation was lagging behind wage growth from 1991 to 2005. Table 1 indicates the correction of the pension base by the valuation coefficient as planned by the 1999 pension legislation (Official Gazette of the Republic of Slovenia, No. 106/99). The valuation coefficient for 2005 is just 0.777, which basically means that someone who has been earning the average wage throughout his or her career and retired in 2006 will have a pension base for calculation of the pension reaching 77.7% of the average wage.

TABLE 1: *Value of the valuation coefficient in real terms at a 2.5% steady state growth*

| Year | Valuation coefficient |
|---|---|
| 2000 | 0.82 |
| **2005** | **0.78** |
| 2010 | 0.76 |
| 2015 | 0.74 |
| 2020 | 0.72 |
| 2025 | 0.71 |
| 2030 | 0.69 |

*Note: Valuation coefficients are calculated such that at a 2.5% steady state growth and 80% indexation of pensions to wages they ensure the horizontal equality of pensioners retiring (under equal entry conditions) under the old Pension Act (OGRS, No. 12/92) and the new Pension Act (OGRS, No. 106/99).*

*Source: Pension and Disability Insurance Act (OGRS, No. 106/99); own calculations and model simulations.*

In terms of calculating the pension base, one can speak of a covert "valuation tax" which reduces the individual's pensionable income for calculation of the pension base (Wiese 2004: 32, 36-47). The shock that economic policy caused to the pension system in 1991 when pensions were not indexed to wage growth was transferred by the 1999 pension reform into the system of calculating the valuation coefficients, which made the method of calculating the pension base, and hence also pensions, very untransparent. In this model the valuation coefficient remains unchanged after 2030 as it is assumed that a continued fall would lead to unacceptably low pensions[4].

[4] If the valuation of the pension base had not been restricted in the model, the tax rate of the valuation tax would have been asymptotically approaching 100% and thus the value of the replacement rate would have been approaching zero. It is assumed that this would lead to changes in pension legislation.

Based on stochastic simulations, Wiese (2004) indicated in the case of the pension indexation that was valid before the change in the method of harmonising pensions in 2005 (Official Gazette of the Republic of Slovenia, No. 72/05), that the pensions index was lagging behind the wage index by 0.5% in over 80% of replications. It can be assumed that between 2000 and 2005 the pension index was equal to the wage index, reduced by 0.5%. The change in the pension legislation in 2005 (Official Gazette of the Republic of Slovenia, No. 72/05) limited the value of the valuation coefficient to 0.777, as specifically indicated in Table 1. Another option is the use of wage indexation in simulations where the wage index is multiplied by the exogenous factor $K_{WI}$. This can be changed appropriately, thus adjusting the calculation of pensions.

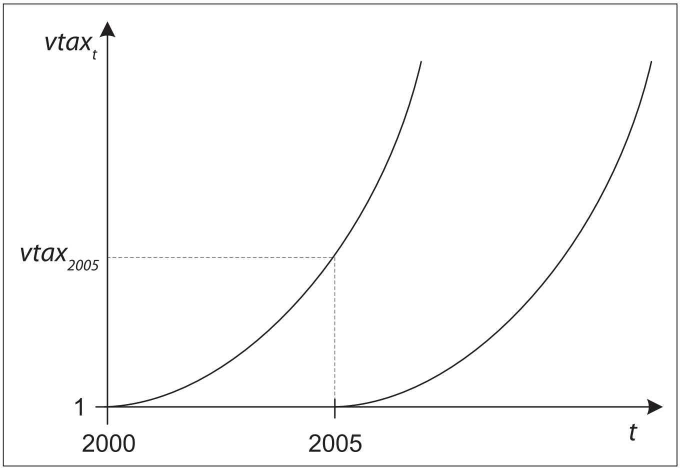
Modelling the pension base is carried out in several parts which are brought together in expression (5). This is required because the system for defining pensions (valuation, accrual and indexation), which is given in Figure 1 is complex and specific to each generation. It is therefore impossible to model the procedures of the pension definition with a sufficient degree of realism and consistency in a generation-independent way, as is possible in some less complex pension systems such as the Swiss system (Müller *et al.* 2003). To this end, the article first defines the pension index for pension calculation, then calculates the pension base and finally corrects the calculated pension base by taking the minimum and maximum pension base into account. The next section indicates the modelling of harmonising pensions with wages.

The pension index for calculation of the pension of generation $g$, $\iota_{t,g}^{rent}$, is given by the following expression:

$$\iota_{t,g}^{rent} = \frac{\mu_{t,g}^{aret} vtax_t}{\sum_v \mu_{v,g}^{ret} vtax_v}, \tag{6}$$

where $\mu_{t,g}^{aret}$ is the multiplier for the generation that has already retired in year $t$, $\mu_{t,g}^{ret}$ is the multiplier for the generation retiring in year $t$ and $vtax_t$ the valuation tax. Figure 2 indicates the generation-specific definition of the pensions mentioned above, which is expressed in the pensions index via the valuation tax. The curve arising from the origin represents the pension profile of the generation retiring in the five-year period ending in 2000. The value of the valuation tax equals one at the starting point, and then changes according to the recursive formula that will be defined in expression (16). The generation retiring in the next period, ending in 2005, has a different pension profile as indicated by the second curve. This is achieved by transforming the first curve for $t \geq 2005$ using the expression $vtax_t \,/\, vtax_{2005}$. The new profile is acquired by setting the original pension profile to 1 at the moment of the transition ($t$ = 2005) and applying a recursive definition of the valuation tax, where the original profile continues along its original path.

FIGURE 2: *Generation-specific pension profiles*




The pension base for calculating the pension of generation *g* (without a correction for the minimal pension base), $pb_{g,h}$, is calculated separately for generations born within the model horizon (newborn generations):

$$pb_{g,h} = \frac{1}{v^{rh}_{g,h}} \sum_{\substack{t \\ (\bar{l}_{t,g,h}>0)\wedge(\mathrm{map}[pb_{t,g}]=1)}} \left( \frac{vtax_t \, p_{el,t,g,h} \, y_{l,t,g,h} \, \bar{\pi}_{t,g,h}}{\bar{p}_t \, \bar{q}_t \, \wp} \right) \tag{7}$$

and for generations already in existence in the model in the first time period (existing generations):

$$pb_{g,h} = \frac{1}{v^{rh}_{g,h}} (1+\gamma)^{-5(12-\mathrm{ord}[g])} \sum_{\substack{t \\ (\bar{l}_{t,0,h}>0)\wedge(\mathrm{map}[pb_{t,0}]=1)}} \left( \frac{vtax_t \, p_{el,t,g,h} \, y_{l,t,0,h} \, \bar{\pi}_{t,0,h}}{\bar{p}_t \, \bar{q}_t \, \wp} \right), \tag{8}$$

where $\bar{l}_{t,g,h}$ is the reference work profile, $\mathrm{map}(pb_{t,g})$ a mathematical operation that assigns the time for calculation of the pension base, $\bar{\pi}_{t,g,h}$ the reference labour productivity profile, $p_{el,t,g,h}$ the price of leisure, and $\wp$ the pension base divisor, including the calibration correction. The pension base divisor in our model with five-year time and age intervals has on principle the value of 3.5, which assumes that the full service period is 17.5 and not 18 years.

Slovenia's pension system does not have an explicit value for the minimum and maximum pension but has a minimum ($pb_t^{min}$) and maximum pension base ($pb_t^{max}$), which (in the model) may be based on the average net or average gross wage $\bar{w}_t$:

$$\begin{aligned} pb_t^{min} &= \vartheta^{min} \bar{w}_t, \\ pb_t^{max} &= 4\, pb_t^{min}, \end{aligned} \tag{9}$$

where the factor $\vartheta^{min}$ equalled 39.4% in 2000 for gross wage or 62.5% for net wage (Majcen *et al*. 2005a: 142). The minimum ( $pens_{t,g}^{min}$ ) and maximum pension ( $pens_{t,g}^{max}$ ) for period $t$ are given by expressions:

$$
\begin{aligned}
pens_{t,g}^{min} &= pb_t^{min}\alpha_{t,g}, \\
pens_{t,g}^{max} &= 4\,pens_{t,g}^{min},
\end{aligned} \tag{10}
$$

while the actual pension for generation $g$, $pens_{t,g}$, based on the minimum pension in the base year, $pens_g^{min}$, is given by this expression:

$$
pens_{t,g} = \max\left[pens_g^{min}, \min\left(4\,pens_g^{min}, pens_{t,g}\right)\right]. \tag{11}
$$

However, this version of the OLG-GE model uses a *max-min* formulation for determining the actual pension directly in the pension base calculation. The pension base for calculating the pension of generation $g$ with a correction for the minimum and maximum pension base, $pb_{g,h}^{mm}$, is arrived at using the expression:

$$
pb_{g,h}^{mm} = \max\left[pb_g^{min}, \min\left(4\,pb_g^{min}, pb_{g,h}\right)\right], \tag{12}
$$

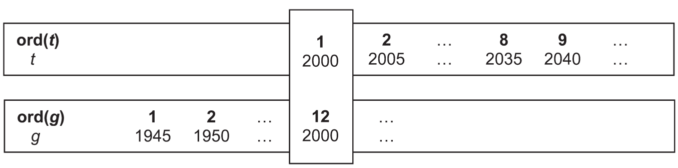
while the minimum pension base of generation $g$, $pb_g^{min}$, is given by the expression:

$$
pb_g^{min} = \sum_{\substack{t \\ \mathrm{ord}[t]=\mathrm{ord}[g]-4}} \left(\iota_t^{min}\, pb^{min}\right), \tag{13}
$$

where $\iota_t^{min}$ is the pensions index for calculating the minimum pension base, and $pb^{\min}$ is the minimum pension base in 2000 (the base year).

The constraint on the product of the pensions index for calculating the minimum pension (pension base) and the minimum pension base in the base year in expression (13), $\mathrm{ord}(t) = \mathrm{ord}(g) - 4$, is required because the model includes a discrepancy between the set of generations $g$ and the set of time periods $t$, where ord($t$) and ord($g$) are mathematical operations that assign numerical values to the elements of sets $t$ and $g$. Namely, the time period set has 2000 (base year) as the first element, while the generation set has 1945 (the base year reduced by the number of annual generations in the model) as the first element. It has to be recalled that the model deals with five-year time periods which are constructed so that the first time period covers years 1996 to 2000, and the first generation covers years 1941 to 1945. Figure 3 presents the background mechanism for the constraint on the product of the pension index.

FIGURE 3: *Time periods and generations in a dynamic general equilibrium framework*



Take as an example the 12th generation which, for the purposes of the model, was born in the first time period[5] (1996-2000). This generation will retire without pension deductions in the ninth time period (2036-2040), while its pension will be calculated based on service years, which will conclude in the eighth time period (2031-2035). This explains the gap between the generation and the time period in the constraint on the product of the pension index presented by expression (13).

### 3.3 Adjustment of Pensions with Respect to Wages

As stated above, there is a choice between the approach used by Wiese (2004) and arbitrary wage indexation in the approximation of the pension index. Instead of using the wage index and pension index, the model therefore only uses the wages index either corrected by 0.5% or multiplied by an exogenous factor. An existing pension is additionally corrected with a value of –0.65% until 2025 due to the adjustment of pensions between existing and new pensioners, which is described in detail by Verbič (2007: 112-122). The approach employed by Wiese (2004: 37), which applies in the case of harmonising pensions that was valid before the 2005 change in the pension legislation (Official Gazette of the Republic of Slovenia, No. 72/05), provides the following pensions index:

$$\iota_t = \begin{cases} \dfrac{w_{v-1}}{w_{v-2}} - 0.005, & \forall v = t+1; \\ \dfrac{w_{v-1}}{w_{v-2}} - 0.005 - 0.0065, & \forall v < t+1 < 2025, \end{cases} \tag{14}$$

while the arbitrary wage indexation that applies independently of the currently valid pension indexation percentages gives the following index:

$$\iota_t = \begin{cases} \dfrac{w_{v-1}}{w_{v-2}} K_{WI,t}, & \forall v = t+1; \\ \dfrac{w_{v-1}}{w_{v-2}} K_{WI,t} - 0.0065, & \forall v < t+1 < 2025, \end{cases} \tag{15}$$

where $K_{WI}$ is the coefficient of adjustment of the growth of pensions with respect to the growth of wages (coefficient of the wage indexation). Approach (15) was employed in this article. The adjustment of pensions between existing and new pensioners is implemented in order to equal the pension levels between former insured persons who have already retired and insured persons retiring now. Without the adjustment the latter group would have a smaller pension due to the falling value of the accrual rate and the valuation coefficient. The pensions of existing pensioners are therefore adjusted downwards. The minimum pension is also adjusted by the pension index and is decreasing relatively over time.

[5] The generation was actually born in the 1976-1980 time period, as generations only appear in the model during their active working life, i.e. after 20 years of age.

In our general equilibrium model, the modelling of this part of the pension system has also been simplified slightly, which is reflected in the adjustment of pensions between existing and new pensioners being absent. The growth in the valuation tax is given in the expression below, which *de facto* presents a recursive definition of the valuation tax:

$$\frac{vtax_{t+1}}{vtax_t} = 1 + \left[ (1+\gamma)(1+\bar{r}) \frac{p_{l,t+1}}{p_{l,t}} - 1 \right] K_{WI,t} , \tag{16}$$

while the pension index for calculation of the minimum pension base, $\iota_t^{min}$, is given by the following expression (Weise, 2004: 47–48):

$$\iota_t^{min} = 1 + \left( \frac{\bar{q}_t}{\bar{p}_t} \frac{p_{l,t}}{p_{l,0}} - 1 \right) K_{WI,t} , \tag{17}$$

where the products $(1+\gamma)(1+\bar{r}) \frac{p_{l,t+1}}{p_{l,t}}$ in expression (16) and $\frac{\bar{q}_t}{\bar{p}_t} \frac{p_{l,t}}{p_{l,0}}$ in expression (17) match the wage growth illustrated in expression (15). In addition to productivity growth, $\bar{q}_t = (1+\gamma)^t$, a correction due to the use of discounted prices is also introduced, $\bar{p}_t = (1+\bar{r})^{-t}$.

## 4. MODELLING THE SECOND PENSION PILLAR

The second pillar of the pension system in Slovenia comprises the supplementary pension insurance which can be broken down into: (1) individual and collective; (2) voluntary and mandatory; and (3) based on employee or employer payments. Following the significant consolidation of the pensions market in the first half of this decade, the first and third classifications can be regarded as being practically the same, and one can speak of *individual supplementary pension insurance* as insurance based on employee payments, and *collective supplementary pension insurance* as insurance based on employer payments. Distinguishing between whether participation is mandatory and the type of scheme is somewhat more difficult; individual supplementary pension insurance is voluntary, while collective supplementary insurance may be mandatory or voluntary. Unfortunately, the data collected do not permit us to distinguish between policyholders in terms of whether their insurance is mandatory or not.

In order to model the second pillar of the Slovenian pension system within this OLG-GE model, the supplementary pension insurance profiles first have to be designed, which is set out in the next section. The subsequent analysis addresses the relationship between the premium paid and the pension paid out from supplementary pension insurance. The key elements used in similar models around the world to capture the second pillar are then assessed to determine their suitability and feasibility in this model. One of the possible methods will finally be selected on the basis of available resources and implemented for the pension block in this OLG-GE model.

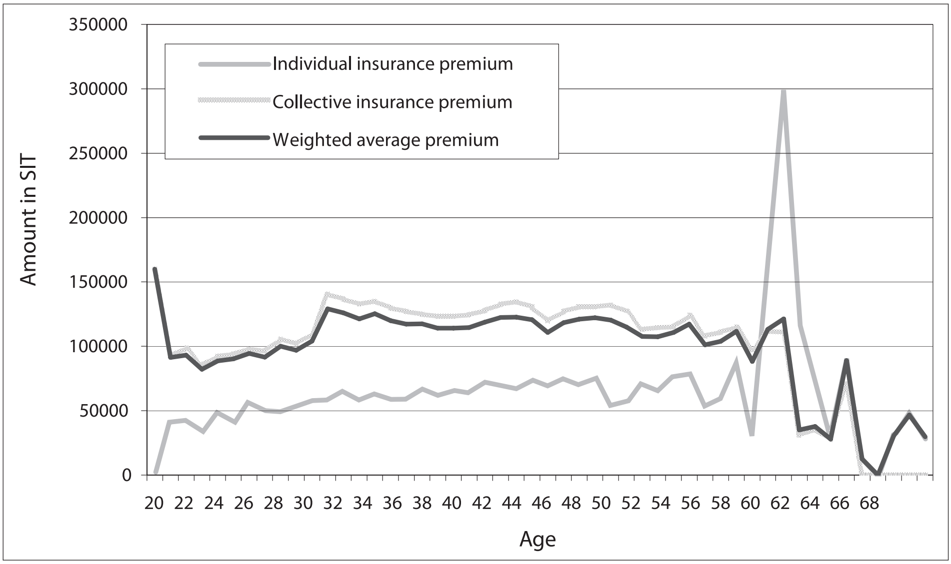
### 4.1 The Supplementary Pension Insurance Profiles

The consistent disaggregated data that would be required for any in-depth analysis of the pension system's second pillar, except some partial attempts to collect and analyse them (Stanovnik 2004a; Slapar 2005; Majcen *et al*. 2006), so far do not exist; therefore they first had to be acquired and processed appropriately. This involves: (1) microdata from the Statistical Office of the Republic of Slovenia (SORS), which were already used to analyse the long-term sustainability of the first pension pillar and the significance of the second and third pillars (Majcen *et al*. 2006); and (2) data from the Insurance Supervision Agency (ISA) model's database, which has already been used to produce the projections of revenues and income of voluntary collective pension insurance (Slapar 2005). The SORS microdata supports a detailed analysis of individual supplementary pension insurance from the policyholder level to the most aggregated forms, while data from the ISA model database supports the analysis of collective supplementary pension insurance from the pension institution level up to the most aggregated forms. Below the article sets out the profiles for supplementary pension insurance in Slovenia, while the procedure for their formulation has already been described in detail in Verbič (2007: 214-221).

In order to formulate a single supplementary pension insurance profile for the Republic of Slovenia, the individual and collective supplementary pension insurance profiles had to be combined. This was not a simple task as there are some policyholders included in both forms of pension schemes, i.e. they appear twice. However, this issue was not specifically addressed in the analysis because of the low number of such cases (Verbič 2007: 218). The supplementary pension insurance premium was calculated as the weighted average of the individual supplementary pension insurance premium and the collective supplementary pension insurance premium, where the number of policyholders in both pension scheme types was used for weighting.

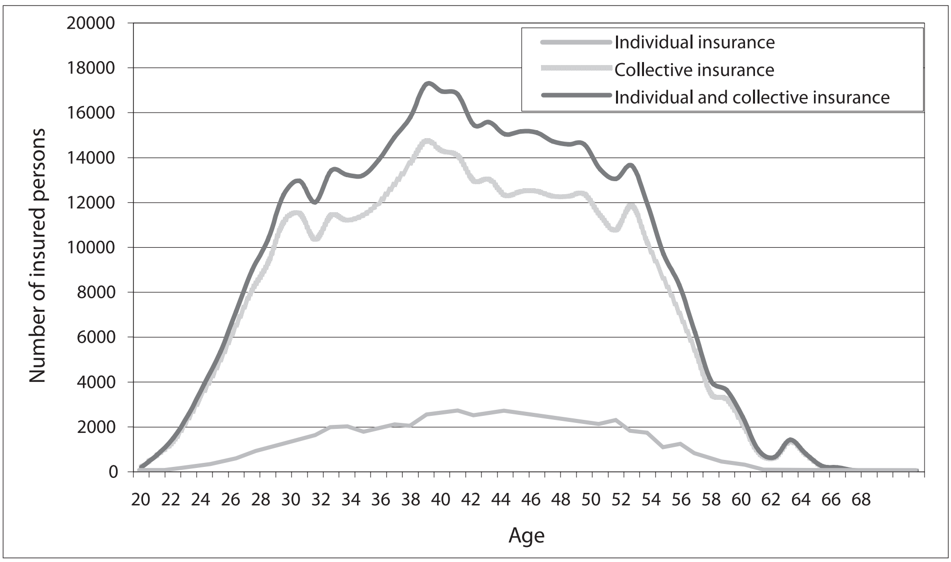
The age structure of the average annual supplementary pension insurance premia in 2004 is given in Figure 4. It can be noted that the supplementary pension insurance premium profile follows the collective supplementary pension insurance premium profile quite closely, although its level is slightly lower. Only the 60-64 and 65-69 age brackets exhibit higher volatility in the individual supplementary pension insurance premium, which causes the supplementary pension insurance premium profile to move slightly above the profile of the collective supplementary pension insurance premium. The age structure of the number of supplementary pension insurance policyholders in 2004 is given in Figure 5. It can be established that the collective supplementary pension insurance policyholders represents the majority of policyholders. The problem of the small number of observations in some age brackets (20-24 and 60-69) of course remains so the calculated premia for supplementary pension insurance in those age brackets have to be considered with some caution.

FIGURE 4: Average annual supplementary pension insurance premium in 2004




*Sources: SORS Microdata (2006) and ISA Model Database (2005); own calculations.*

FIGURE 5: *Number of insured persons of supplementary pension insurance in 2004*

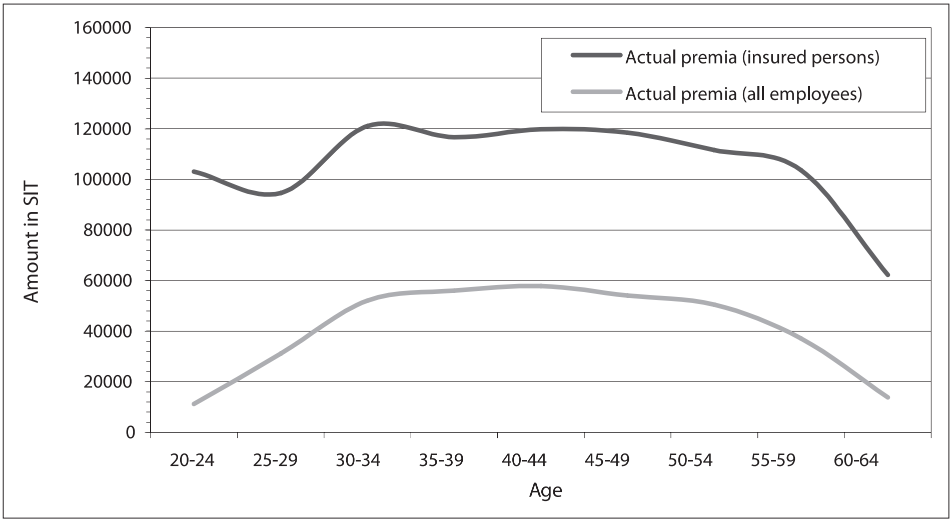



*Sources: SORS Microdata (2006) and ISA Model Database (2005); own calculations.*

Based on the results illustrated in Figures 4 and 5, one can conclude that the dimensions of individual supplementary pension insurance in Slovenia for the type of analysis in this article, i.e. an analysis using the dynamic general equilibrium model for the Slovenian economy, are negligible in practical terms. Use will therefore only be made below of a single, unified supplementary pension insurance profile.

Further, in modelling the second pillar of the Slovenian pension system the profile of the average employee in terms of their participation in supplementary pension insurance will be of greater interest to this article than the profile of the average supplementary pension insurance policyholder in the Republic of Slovenia. For this reason, the age structure of supplementary pension insurance policyholders is replaced by the age structure of employees to acquire the actual average annual premium in 2004, which is represented by five-year age brackets in Figure 6.

FIGURE 6: *Average annual supplementary pension insurance premium in 2004, taking into account insured persons only and all employees*




*Sources: SORS Microdata (2006) and ISA Model Database (2005); own calculations.*

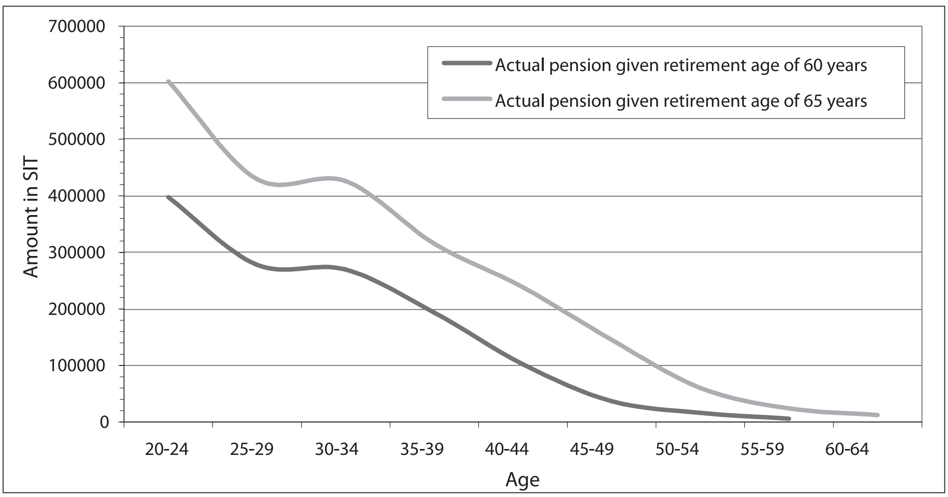
It can be seen that the actual average premium in 2004, taking into account all employees, is significantly lower than the actual average premium for supplementary pension insurance policyholders as only approximately half of all employees are included in that form of insurance, and almost 40% of that group are civil servants for whom only the minimum supplementary pension insurance premium is being paid for by the government. It can be seen that the supplementary pension insurance premium profile for all employees first grows, is relatively stable for employees between 35 and 50 years of age, and then starts to fall. The supplementary pension insurance premium profile that only relates to policyholders does not indicate this specific pattern of growth in the initial period of employment service, although after 50 years of age the trend for policyholders matches the profile for employees.

### 4.2 The Premium and the Supplementary Pension

The link between the premium paid and the pension paid out from supplementary pension insurance is of key significance to the analysis of supplementary pension insurance. Primarily of interest is the level of pension that would be paid out from supplementary pension insurance based on the premia actually paid in Slovenia determined for 2004 by the construction of supplementary pension insurance profiles. To this end, a modelling tool based on the contribution by Majcen *et al*. (2006) was used. The International Labour Organisation (ILO) family of models was used as the framework for the calculation and simulation of macroeconomic categories, and the simulation of the current pension legislation, while use is also made in tandem with the intergenerational accounting model and the annuity calculation based on assumptions on the required amount of supplementary pension saving (Verbič 2007: 221-226).

The annual value of the pension from supplementary pension insurance that an individual would receive in the first full year after their retirement is calculated first on the basis of the actual average supplementary pension insurance premium paid by policyholders only, and then for all employees in 2004, as illustrated in Figure 6. The results, broken down into five-year age brackets, are given in Figures 7 and 8, based on the assumption of two different retirement ages.

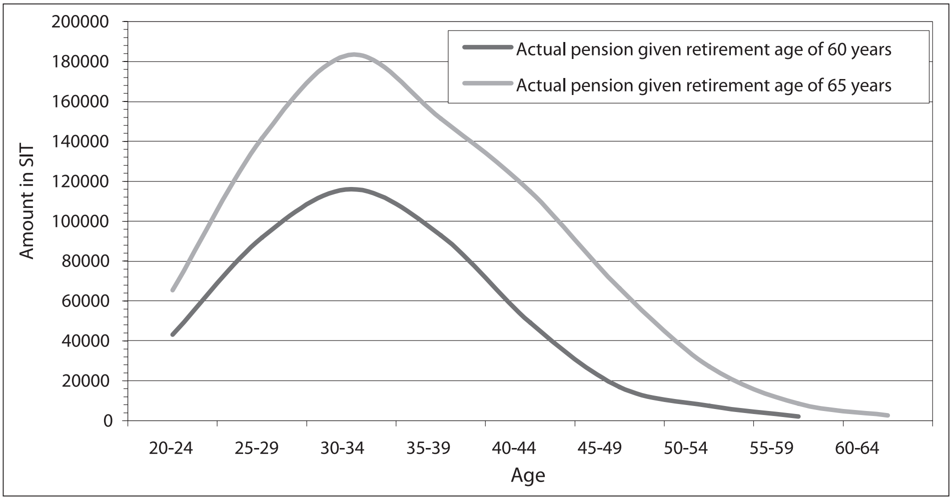
FIGURE 7: *Supplementary pension at retirement based on paid supplementary pension insurance premia for insured persons in 2004*




*Sources: Own model simulations based on the model of Majcen et al. (2006); own calculations on the basis of SORS Microdata (2006) and ISA Model Database (2005).*

It can be seen that a policyholder's pension (Figure 7), calculated for the first year after their retirement, decreases quite sharply with age. The given assumptions anticipate this as older policyholders have fewer years left to save until their retirement, which means less accumulated funds for calculating the pension annuity. Assuming the retirement age of 65 years, an individual has five more years available for saving that with an assumed retirement age of 60 so the supplementary pension curve is higher for the first version than the supplementary pension curve in the second version, although with increasing age the two curves converge. The convergence can be seen between the curves themselves and in the tendency of both curves toward a null pension. The only break in the fall of pensions with age occurs in the 25-29 age bracket, where – as seen in Figure 4 – there was growth in the actual average supplementary pension insurance premium compared to the preceding five-year age bracket.

FIGURE 8: *Supplementary pension at retirement based on paid supplementary pension insurance premia for all employees in 2004*




*Sources: Own model simulations based on the model of Majcen et al. (2006); own calculations on the basis of SORS Microdata (2006) and ISA Model Database (2005).*

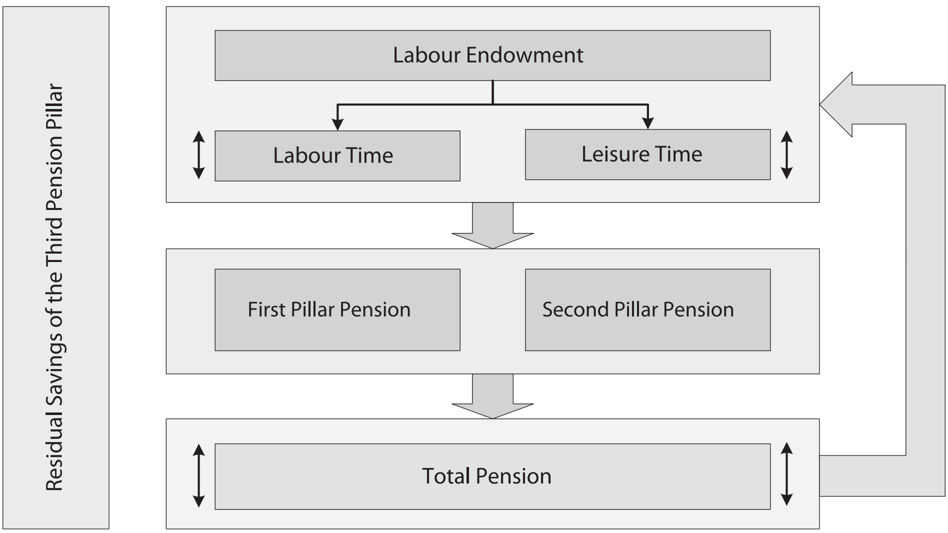
The pension calculated for the first year after retirement on the basis of actual premia and taking into account all employees (Figure 8) has a different age structure from the pensions for which only supplementary pension insurance policyholders were taken into account. It can be seen that, due to the growing proportion of policyholders in the total number of employees, the pension calculated on the basis of actually paid premia first rises, peaking between the ages of 30 and 35, before decreasing. As may be seen from a comparison of Figures 7 and 8, the characteristic trends arising from the different retirement ages are retained.

### 4.3 The Liquidity Constraint and Supplementary Pension Savings

A review of overlapping-generations general equilibrium modelling (Verbič 2007: 22–55) indicates that the field is so new and complex that to date only a handful of more or less successful attempts to model the second pension pillar have been made. At present we are aware of three such attempts, which are listed herein by descending complexity of the pension block within the OLG-GE framework, but also by ascending relevance in terms of the institutional characteristics of the Slovenian pension system (Verbič 2007: 227–235). These are the model of the Danish economy by Knudsen *et al*. (1998), the model of the Dutch economy by Draper *et al*. (2005), and the model of the Lithuanian (Lassila 1999) and Finnish economies (Alho *et al*. 2006).

Based on the review of existing modelling of supplementary pension insurance in models of this kind, and the resources available, it was decided to introduce a liquidity constraint to the SIOLG 2.0 model. This approach is similar to that employed in Lassila's (1999) model of the Lithuanian economy. To this end, the category of total pension was introduced, comprising the pension from the first pension pillar and the pension from the second pension pillar, while saving in the third pillar of pension insurance remains residual and is not explicitly modelled. The functioning of the pension system, as modelled in the model SIOLG 2.0, is illustrated in Figure 9.

FIGURE 9: *Modelling mandatory and supplementary pension insurance*




Every household decides on the use of its labour endowment as either labour or leisure. It earns a net wage based on the labour time, on which labour tax is then paid. The net wage and the amount of labour tax, roughly speaking, comprise the gross wage from which social contributions are paid, including contributions for mandatory pension insurance. The pension from the first pension pillar is being calculated on the basis of the gross wage. On the other hand, households also save within the second pension pillar in accordance with the supplementary pension insurance profiles. This (largely) involves supplementary pension insurance within the voluntary second pillar of the pension system. The actuarial calculation of the second pillar pension is made on the basis of the premia paid. The sum of the pension from the first and second pillars of the pension insurance therefore represents the total pension.

On the other hand, households can also decide on the total pension they will receive after retirement. To this end, they adjust the ratio between their labour and leisure time in order to meet their objectives throughout their active working life (or the remainder thereof). If a household after retirement wants a pension higher than the reference pension it has to increase its activity by increasing labour time at the expense of leisure time, given the fixed labour endowment, $\omega_{t,g,h} = 1$. On the other hand, the household may also reduce its consumption of goods and services, which is not illustrated in Figure 9. When the sum of the total target pension is defined outside the model, i.e. exogenously, one can speak of mandatory supplementary pension insurance or the mandatory second pillar. Due to the assumptions of the model regarding the rationality and perfect foresight of households, their decision-making is therefore constrained and their decisions are potentially less optimal, while the welfare level is lower. However, the liquidity constraint also changes the macro-economic results, which can lead to some interesting overall effects.

The value of the liquidity constraint, $liqcons_{g,h}$, is a shadow price that defines the portion of the net wage allocated to saving within the second pillar of the pension system. The actual form of the liquidity constraint depends on the counterfactual scenario relating to the second pension pillar (Verbič 2007: 236, 244–246). The liquidity constraint, which only models the existing second pension pillar and allows full effects of the pension reform, is relatively simple:

$$liqcons_{g,h} = \frac{v^{rh}_{g,h}\theta^{\#}_{g,h}}{\Im_g \sum_t \left( y_{l,t,g,h} \dfrac{p_{el,t,g,h}}{\overline{p}_t} \overline{\pi}_{t,g,h} \right)} \sum_t \left[ \mu^{aret}_{t,g} p_f \overline{spillar}_g \sum_v \left( \mu^{ret}_{v,g} pens_{v,g,h} \right) \right], \tag{18}$$

while the liquidity constraint, which in the reference scenario keeps saving in the second pillar of the pension system at the existing level, but in the counterfactual scenario (partially) compensates for the pension reform, has the following form:

$$\begin{aligned} liqcons_{g,h} = {} & \frac{\left(1-\mu^{sp}\right) v^{rh}_{g,h}\theta^{\#}_{g,h}}{\Im_g \sum_t \left( y_{l,t,g,h} \dfrac{p_{el,t,g,h}}{\overline{p}_t} \overline{\pi}_{t,g,h} \right)} \sum_t \left[ \mu^{aret}_{t,g} p_f \overline{spillar}_g \sum_v \left( \mu^{ret}_{v,g} pens_{v,g,h} \right) \right] + \\ & + \frac{\mu^{sp} v^{rh}_{g,h}\theta^{\#}_{g,h}}{\Im_g \sum_t \left( y_{l,t,g,h} \dfrac{p_{el,t,g,h}}{\overline{p}_t} \overline{\pi}_{t,g,h} \right)} \sum_t \left[ \mu^{aret}_{t,g} p_f \left( \frac{\tilde{\alpha}_g}{\alpha_g} \tilde{\imath}_t \sum_v \left( \mu^{ret}_{v,g} pens_{v,g,h} \right) - pens_{t,g,h} \right) \right], \end{aligned} \tag{19}$$

where $\mu^{sp}$ is the multiplier for counterfactual scenarios with an active second pillar, $\Im_g$ is the ratio of interest-bearing supplementary pension insurance savings to non-interest-bearing savings, $\theta^{\#}_{g,h}$ the number of adults in a household, $v^{rh}_{g,h}$ is the number of retired households, $y_{l,t,g,h}$ is the level of the labour supply, $\overline{\pi}_{t,g,h}$ is the reference labour productivity profile, $p_{el,t,g,h}$ is the price of leisure, $\overline{p}_t$ is the reference steady state price, $p_f$ is the price of foreign currency, $\mu^{aret}_{t,g}$ is the multiplier for the generation already retired in year $t$, $\mu^{ret}_{t,g}$ is the multiplier for the generation retiring in year $t$, $pens_{t,g,h}$ is the pension of each retired household, $\alpha_g$ is the actual accrual rate for calculating the pension of generation $g$, $\tilde{\alpha}_g$ is the target accrual rate for calculating the pension of generation $g$, $\tilde{\iota}_t$ is the generation-independent pension indexation factor, and $\overline{spillar}_g$ is the pension from supplementary pension insurance as a proportion of the pension from mandatory pension insurance in the reference scenario.

In order to achieve the various objectives of the analysis, the parameters of liquidity constraint (19) are varied within the framework of given counterfactual scenarios. The key supplementary pension insurance parameters to be adjusted are: (1) the target accrual rate for pension calculation, $\tilde{\alpha}_g$, i.e. the target proportion of the total pension in the pension base; and (2) the multiplier for the generation retiring in a given year, $\mu^{ret}_{t,g}$, i.e. the period to which the target proportion of the total pension relates.

Now, let us consider the liquidity constraint (19) in greater detail. The first part of the liquidity constraint relates to the reference scenario which models saving in the second pillar of the pension system at the existing level. Reference saving in the second pillar is expressed as a proportion of the first pillar pension in the first year following retirement. The latter has the form $\sum_v \left(\mu^{ret}_{v,g} pens_{v,g,h}\right)$, where the multiplier $\mu^{ret}_{v,g}$ has only one non-zero element for the retirement year, therefore the just presented sum also contains only one non-zero element. The age structure of the pension from the second pension pillar, calculated on the basis of premia paid by all employees as a share of the pension from the first pension pillar in the first year after retirement, $\overline{spillar}_g$, is independent of the statutory retirement age, as illustrated in Figure 8.

Aggregation by index $t$ gives us the existing savings in the second pillar of the pension system for retired generations (with multiplier $\mu^{aret}_{t,g}$). The latter is then aggregated based on the number of pensioners, $\underline{v}^{rh}_{g,h}\theta^{\#}_{g,h}$, and appropriately discounted using the reference steady state price, $\overline{p}_t$. In order to acquire the portion of net wages allocated to saving within the second pillar of the pension system, the calculated savings are further divided by the net income from employment, $\sum_t \underline{y}_{l,t,g,h} p_{el,t,g,h} \pi_{t,g,h}$. As capital is not explicitly modelled in our OLG-GE model, the compounding mechanism has to be approximated in case of supplementary pension saving accumulation as otherwise there would be a growing gap between interest-bearing supplementary pension insurance savings to which the compounding mechanism applies and the non-interest-bearing supplementary pension insurance savings, where only paid premia are aggregated. The correction is presented by the ratio of interest-bearing to non-interest-bearing supplementary pension insurance savings, $\Im_g$, defined as follows:

$$\Im_g = \frac{\left[\left(1+\overline{r}\right)^{N_{w,g}} - 1\right]\left(1+\overline{r}\right)}{\overline{r}N_{w,g}}, \tag{20}$$

where $\underline{N}_{w,g}$ is the average number of years of service for an individual generation calculated on the basis of the five-year intervals, and $\overline{r}$ (reference) annual interest rate.

The second part of the liquidity constraint relates to the selected counterfactual scenario which models the saving in the second pension pillar required to achieve the target total pension. Hypothetical saving in the second pillar is expressed as the difference between the target total pension and the actual first-pillar pension. The target total pension is arrived at by correcting the first-pillar pension in the first year after retirement, $\sum_v \left(\mu^{ret}_{v,g} pens_{v,g,h}\right)$, by the ratio between the target and actual accrued rate, which amounts to $\tilde{\alpha}_g \alpha_g^{-1}$, and the value obtained is then indexed to wages using the pension indexation factor $\tilde{\iota}_t$. The latter is required because the actual pension is indexed, while the target pension would otherwise not have been. Due to the multiplier $\mu^{ret}_{v,g}$ the sum above only has one element so the pension index for calculating pensions $\iota^{rent}_{t,g}$, used in expression (5) is no longer having an effect over time. The generation-independent pension indexation factor, $\tilde{\iota}_t$, is given by the expression:

$$\tilde{\iota}_t = \frac{\iota^{min}_t}{\sum_v \mu^{ret}_{v,g} \iota^{min}_v}, \tag{21}$$

where $\mu^{ret}_{t,g}$ is the multiplier for the generation retiring in year $t$ and $\iota^{min}_t$ is the generation-independent pension index, which was also used to calculate the minimum pension (minimum pension base) of generation $g$.

The differences between the target total pension and the actual first pillar pension is then aggregated by index $t$ for retired generations (with multiplier $\mu^{aret}_{t,g}$), which gives the required total saving in the second pension pillar. The latter is then again aggregated based on the number of pensioners, $\nu^{rh}_{g,h}\,\theta^{\#}_{g,h}$, and appropriately discounted using the reference steady state price, $\overline{p}_t$. In order to acquire the portion of net wages allocated to saving within the second pension pillar, the calculated savings are further divided by the net income from employment, $\sum_t y_{l,t,g,h} p_{el,t,g,h} \overline{\pi}_{t,g,h}$, and an approximation of the compounding mechanism is carried out on accumulated assets by using the ratio between the interest-bearing and non-interest-bearing supplementary pension savings, $\Im_g$. Since already retired generations have a null value of the liquidity constraint, the liquidity constraint divisor in the model has to be adjusted, to avoid a division by zero.

This concludes the modelling of the second pension pillar in our OLG-GE model. In the reference scenario there is always only the existing second pension pillar that is being modelled. From the technical point of view, this is mandatory pension saving but since in so doing we are modelling the existing second pillar saving, which is (mainly) voluntary, it can be understood as a voluntary second pillar. In the first counterfactual scenario, which follows from expression (18), the extent of the second pillar does not change, while in the set of counterfactual scenarios arising from expression (19) the extent of the second pillar is adjusted to the target total pension. In that case, the total saving required in the second pillar may be calculated, which can only be understood as a mandatory second pension pillar. Additional saving in the second pillar is therefore the difference between the required level of saving in the counterfactual scenario and the existing level of saving in the reference scenario.

## 5. CONCLUSION

This article presents an important upgrade of the overlapping-generations general equilibrium model with a pension system. Within the pension block we model both the first pension pillar, financed on a pay-as-you-go basis, and the fully-funded second pillar of the Slovenian pension system. At present in Slovenia the third pension pillar does not even have an appropriate legal basis so it is considered as residual saving in our model. The modelling of the first pension pillar was designed to capture the key pension system parameters that are usually the subject of change within pension reforms. Our work focused on cash flows of the mandatory pension insurance institution (revenues and expenditure of the Institute of Pension and Disability Insurance), the relationship between the pension base and the pension, and the process of harmonising pension growth with wage growth.

The modelling of the second pension pillar provided a greater challenge as the review of overlapping-generations general equilibrium modelling already indicated that the field is so new and complex that to date only a handful of more or less successful attempts to model the second pension pillar exists. Based on the available resources it was decided to model supplementary pension insurance with a focus on the implementation of a liquidity constraint. Supplementary pension profiles were created and the relationship between premia paid and pensions paid out from supplementary pension insurance was studied. The category of total pension was introduced, representing the sum of the pension from the first and second pillars, and the model ensured that at every point households adjusted their labour supply and their current consumption towards the target total pension. This creates a certain amount of supplementary pension saving which can be treated as mandatory supplementary pension insurance if the target total pension is defined at a level that differs from the reference level.

Of course, we are aware that economic models are merely tools intended to replicate and analyse a specific economic theory or a part thereof, and as such are always an incomplete and deficient representation of reality. The same applies to our dynamic overlapping-generations general equilibrium model of the Slovenian economy. However, in terms of its capacity to capture the socio-economic reality and in terms of available levels of socio-economic analysis it can be realistically assessed that at present there is no better or more complete deterministic instrument to meet the objectives set herein than a dynamic general equilibrium model.

RECEIVED: MAY 2007

## REFERENCES

Aglietta, M., J. Chateau, J. Fayolle, M. Juillard, J. Le Cacheux, G. Le Garrec and V. Touzé (2007), "Pension Reforms in Europe: An Investigation with a Computable OLG World Model", *Economic Modelling* 24 (3), 481–505.

Alho, J. M., J. Lassila and T. Valkonen (2006), *Longevity Adjustment as a Pension Policy Instrument* Paper Presented at the 62nd Congress of the International Institute of Public Finance, 28-31 August 2006, Paphos: International Institute of Public Finance.

Armington, P. S. (1969), *A Theory of Demand for Products Distinguished by Place of Production* IMF Staff Paper No. 16, Washington, DC: International Monetary Fund.

Arrow, K. J. and G. Debreu (1954), "Existence of an Equilibrium for a Competitive Economy", *Econometrica* 22 (3), 265–290.

Auerbach, A. J. and L. J. Kotlikoff (1987), *Dynamic Fiscal Policy*, Cambridge, NY: Cambridge University Press.

Balistreri, E. J. and R. H. Hillberry (2003), *Estibration: An Illustration of Structural Estimation as Calibration* Mimeo, Golden, CO: Colorado School of Mines.

Böhringer, C., T. F. Rutherford and W. Wiegard (2003), *Computable General Equilibrium Analysis: Opening a Black Box* Discussion Paper No. 56, Mannheim: Zentrum für Europäische Wirtschaftsforschung.

Brooke, A., D. Kendrick, A. Meeraus and R. Raman (1998), *GAMS: A User's Guide*, Washington, DC: GAMS Development Corporation.

Deaton, A. (1992), *Understanding Consumption*, Oxford: Clarendon Press.

Diamond, P. A. (1965), "National Debt in a Neoclassical Growth Model", *American Economic Review* 41, 1126–1150.

Draper, N., A. Nibbelink, J. Bonenkamp and R. Rosenbrand (2005), *Description Gamma Model: Version Pension Study* CPB Memorandum No. 115, Den Haag: Centraal Planbureau.

Equipe Ingénue (2001), *INGENUE: Une modélisation intergénérationnelle et universelle*, Paris: Banque de France.

Equipe Ingénue (2001a), *Macroeconomic Consequences of Pension Reforms in Europe: An Investigation with the INGENUE World Model* CEPII Working Paper No. 17, Paris: Centre d'Etudes Prospectives et d'Informations Internationales.

Equipe Ingénue (2007), *INGENUE 2: A Long Term Intertemporal World Model for the 21st Century* Mimeo, Paris: Centre d'Etudes Prospectives et d'Informations Internationales.

Ferris, M. C. and T. S. Munson (2000), *GAMS/PATH User Guide: Version 4.3*, Washington, DC: GAMS Development Corporation.

Fougère M. and M. Mérette (2000), "Population Ageing, Intergenerational Equity, and Growth: An Analysis with an Endogenous Growth Overlapping Generations Model", In G. Harrison, S. E. H. Jensen, L. H. Pedersen and T. F. Rutherford, eds. *Using Dynamic General Equilibrium Models for Policy Analysis* Amsterdam: North Holland, 325-361.

Friedman, M. (1957), *A Theory of the Consumption Function*, Princeton: Princeton University Press.

Hviding, K. and M. Mérette (1998), *Macroeconomic Effects of Pension Reforms in the Context of Ageing Populations: Overlapping Generations Model Simulations for Seven OECD Countries* OECD Working Paper No. 201, Paris: Organisation for Economic Co-operation and Development, Economics Department.

Knudsen, M. B., L. H. Pedersen, T. W. Petersen, P. Stephensen and P. Trier (1998), *Danish Rational Economic Agents Model – DREAM, Version 1.2* DREAM Documentation, Kobenhavn: Danmarks Statistik.

Lassila, J. (1999), *An Overlapping-Generations Simulation Model for the Lithuanian Economy* ETLA Discussion Paper No. 669, Helsinki: The Research Institute of the Finnish Economy.

Majcen, B., R. van Nieuwkoop, M. Verbič, A. Müller, T. Stanovnik, J. Sambt, M. Čok and F. Kuzmin (2005), *Ekonomske posledice spreminjanja demografske strukture slovenskega prebivalstva (Economic Consequences of Changing Demographic Structure of the Slovenian Population),* Ljubljana: Institute for Economic Research.

Majcen, B., M. Čok, M. Košak and J. Sambt (2006), *Dolgoročna vzdržnost prvega pokojninskega stebra ter pomen drugega in tretjega stebra* (Long-term Sustainability of the First Pension Pillar and the Significance of the Second and Third Pillar), Ljubljana: Institute for Economic Research.

Mathiesen, L. (1985), "Computation of Economic Equilibria by a Sequence of Linear Complementarity Problems", In A. S. Manne, ed. *Economic Equilibrium: Model Formulation and Solution* Amsterdam: North-Holland, 144–162.

Modigliani, F. and R. Brumberg (1954), "Utility Analysis and the Consumption Function: An Interpretation of Cross-Section Data", In K. K. Kurihara, ed. *Post Keynesian Economics* New Brunswick, NJ: Rutgers University Press, 388-436.

Modigliani, F. (1986), "Life Cycle, Individual Thrift, and the Wealth of Nations", *American Economic Review* 76 (3), 297–313.

Müller, A., R. van Nieuwkoop and C. Lieb (2003), *Analyse der Finanzierungsquellen für die AHV: SWISSOLG – ein Overlapping Generations Model für die Schweiz*, Bern: Ecoplan.

Rasmussen, T. N. and T. F. Rutherford (2004), "Modeling Overlapping Generations in a Complementarity Format", *Journal of Economic Dynamics and Control* 28 (7), 1383–1409.

Rutherford, T. F. (1995), *Constant Elasticity of Substitution Functions: Some Hints and Useful Formulae* Mimeo, Boulder, CO: University of Colorado.

Rutherford, T. F. (1995a), "Extensions of GAMS for Complementarity Problems Arising in Applied Economic Analysis", *Journal of Economic Dynamics and Control* 19 (8), 1299–1324.

Samuelson, P. A. (1958), "An Exact Consumption-loan Model of Interest With or Without the Social Contrivance of Money", *Journal of Political Economy* 66, 467–482.

Slapar, M. (2005), "Projekcija izdatkov in prihodkov prostovoljnih kolektivnih pokojninskih zavarovanj" (Projection of Expenditure and Revenue of the Voluntary Collective Pension Insurance), *IB revija* 39 (3), 104–111.

Stanovnik, T. (2004), *The Pension System of Slovenia After the 1999 Reform: Developments and Challenges* Paper Prepared for the Tripartite Conference: Recent Developments in Pension Restructuring in Central and Eastern Europe, 9-10 December 2004, Budapest: International Labour Organization.

Verbič, M., B. Majcen and R. van Nieuwkoop (2006), "Sustainability of the Slovenian Pension System: An Analysis with an Overlapping-generations General Equilibrium Model", *Eastern European Economics* 44 (4), 60–81.

Verbič, M. (2007), *Dinamični model splošnega ravnovesja slovenskega gospodarstva s poudarkom na pokojninskem sistemu* (A Dynamic General Equilibrium Model of the Slovenian Economy with Emphasis on the Pension System), Ljubljana: University of Ljubljana, Faculty of Social Sciences.

Verbič, M. (2007a), "Varying the Parameters of the Slovenian Pension System: An Analysis with an Overlapping-Generations General Equilibrium Model", *Post-Communist Economies* 19 (4), 449–470.

Wiese, P. (2004), *An Evaluation of the Fiscal Stability of Slovenia's Pay-As-You-Go Pension System* Mimeo, Washington, DC: World Bank.

## APPENDICES

### Appendix 1: Overview of Equations of the Model SIOLG 2.0

#### *Zero-profit conditions*

Zero-profit condition of intertemporal utility of the composite consumption:

$$\sum_t \overline{z}_{t,g,h}\,\overline{p}_{z,t,g,h}\left(\frac{p_{z,t,g,h}}{\overline{p}_{z,t,g,h}}\right)^{1-\frac{1}{\theta}} = p_{u,g,h}^{1-\frac{1}{\theta}}\,\overline{PV}_{z,g,h}\,. \tag{A1}$$

Zero-profit condition of the aggregate production:

$$\frac{p_{va,t}}{\overline{p}_t}\frac{\overline{p}_{va,0}\overline{va}_0}{\overline{p}_{y,0}\overline{y}_0}+\sum_{sc}\left(\frac{id_{t,sr,sc}}{\overline{p}_t}\frac{\overline{p}_{A,0,s}\overline{id}_{0,sr,sc}}{\overline{p}_{y,0}\overline{y}_0}\right)=\frac{p_{y,t}}{\overline{p}_t}\,. \tag{A2}$$

Zero-profit condition of the value added:

$$\frac{\overline{l}_{0,s}}{\overline{y}_0}\left(\frac{p_{l,t}}{\overline{p}_t}\right)^{1-\sigma_{kl}}+\left(1-\frac{\overline{R}_{0,s}}{\overline{y}_0}\right)\left(\frac{p_{r,t}\left(1+t_r+\tau_{r,t}\right)}{\overline{p}_t\left(1+\overline{t}_r\right)}\right)^{1-\sigma_{kl}}=\left(\frac{p_{va,t,s}}{\overline{p}_{va,t,s}}\right)^{1-\sigma_{kl}}. \tag{A3}$$

Zero-profit condition of the domestic production:

$$\overline{X}_0 p_f^{1+\eta_X}+\left(\overline{y}_0-\overline{X}_0\right)\left(\frac{p_{H,t}}{\overline{p}_t}\right)^{1+\eta_X}=\overline{y}_0\left[\frac{p_{y,t}}{\overline{p}_t}\right]^{1+\eta_X}. \tag{A4}$$

Zero-profit condition of the Armington good:

$$\left(\frac{p_{A,t}}{\overline{p}_t}\right)^{1-\sigma_A}=\frac{\left(\overline{y}_0-\overline{X}_0\right)}{\left(\overline{c}_0+\overline{g}_0+\sum_{sr}\overline{id}_{0,sr,sc}+\overline{inv}_0\right)}\left(\frac{p_{H,t}}{\overline{p}_t}\right)^{1-\sigma_A}+\frac{\overline{M}_0 p_f^{1-\sigma_A}}{\left(\overline{c}_0+\overline{g}_0+\sum_{sr}\overline{id}_{0,sr,sc}+\overline{inv}_0\right)}. \tag{A5}$$

Zero-profit condition of the capital stock:

$$\overline{k}_0 p_{k,t}=p_{r,t}\overline{R}_0+p_{k,t+1}\overline{k}_0\left(1-\delta\right). \tag{A6}$$

Zero-profit condition of the investment:

$$p_{inv,t}=p_{k,t+1}\,. \tag{A7}$$

Zero-profit condition of the labour market:

$$p_{el,t,g,h}\left(1+t_l\right)\left(1+t_l^{rent}+\tau_{l,t}^{rent}\right)\geq p_{l,t}\overline{\pi}_{t,g,h}. \tag{A8}$$

Zero-profit condition of the total consumption:

$$\alpha_h \left[ \frac{p_{c,t,g,h}\left(1+\tau_{c,t}\right)}{\overline{p}_{c,t,g,h}} \right]^{1-\sigma_{cl}} + \left(1-\alpha_h\right) \left[ \frac{p_{el,t,g,h}}{\overline{p}_{el,t,g,h}} \right]^{1-\sigma_{cl}} = \left( \frac{p_{z,t,g,h}}{\overline{p}_{z,t,g,h}} \right)^{1-\sigma_{cl}} . \tag{A9}$$

Zero-profit condition of the private consumption:

$$\sum_s \beta_{h,s} \left( \frac{p_{A,t,s}(1+t_{VAT,s}+\tau_{VAT,t})}{\overline{p}_t (1+\overline{t}_{VAT,s})} \right)^{1-\sigma_{cc}} = \left( \frac{p_{c,t,g,h}}{\overline{p}_t} \right)^{1-\sigma_{cc}} . \tag{A10}$$

### *Market-clearing conditions*

Market-clearing condition of the Armington good:

$$\begin{aligned} y_{A,t}\left(\overline{y}_0 - \overline{X}_0 + \overline{M}_0\right) = \sum_s \sum_h \sum_g \left[ \overline{c}_{t,g,h,s} y_{c,t,g,h} \left( \frac{p_{c,t,g,h}}{\overline{p}_{c,t,g,h}} \frac{\overline{p}_{A,t,s}(1+\overline{t}_{VAT,s})}{p_{A,t,s}(1+t_{VAT,s}+\tau_{VAT,t})} \right)^{\sigma_{cc}} \right] + \\ + y_{inv,t}\overline{inv}_0 + \sum_s \left( p_{A,t,s}\overline{g}_{0,s} y_{G,t} \frac{1+\overline{t}_{VAT,s}}{p_{A,t,s}\left(1+t_{VAT,s}+\tau_{VAT,t}\right)} \right). \end{aligned} \tag{A11}$$

Market-clearing condition of the labour endowment:

$$\omega_{t,g,h} = y_{l,t,g,h} + \overline{ls}_{t,g,h} y_{z,t} \left( \frac{p_{z,t,g,h}}{\overline{p}_{z,t,g,h}} \frac{\overline{p}_{el,t,g,h}}{p_{el,t,g,h}} \right)^{\sigma_{cl}} . \tag{A12}$$

Market-clearing condition of the labour market:

$$\sum_g \sum_h y_{l,t,g,h} \overline{\pi}_{t,g,h} = \sum_s \left\{ \overline{l}_{0,s} y_{va,t,s} \left( \frac{p_{va,t,s}}{p_{l,t}} \right)^{\sigma_{kl}} \right\} . \tag{A13}$$

Market-clearing condition of capital services:

$$y_{k,t}\overline{R}_0 = \overline{R}_0 y_t \frac{p_{y,t}\left(1+\overline{t}_r\right)}{p_{r,t}\left(1+t_r+\tau_{r,t}\right)} . \tag{A14}$$

Market-clearing condition of the capital stock:

$$y_{k,t-1}\overline{R}_0\left(1-\delta\right) + y_{inv,t-1}\overline{inv}_0 = y_{k,t}\overline{R}_0 . \tag{A15}$$

Market-clearing condition of the domestic supply:

$$y_t \left( \frac{p_{H,t}}{p_{y,t}} \right)^{\eta_X} = y_{A,t} \left( \frac{p_{A,t}}{p_{H,t}} \right)^{\sigma_A} . \tag{A16}$$

Market-clearing condition of the foreign currency:

$$\sum_t \left\{ \overline{X}_0 y_t \overline{p}_t \left( \frac{\overline{p}_t p_f}{p_{y,t}} \right)^{\eta_X} \right\} + \sum_t \sum_g \sum_h \overline{p}_t \zeta_{t,g,h} + \overline{p}_t \overline{q}_t \overline{\zeta}_{IPDI,0} + \\ + \sum_g \sum_h (1-\theta_c) \overline{cma}_{0,g,h} + \sum_t \overline{p}_t \overline{q}_t \left( \overline{D}_0 - \overline{TR}_0 \right) = \\ = \sum_t \left\{ \overline{M}_0 y_{A,t} \overline{p}_t \left( \frac{p_{A,t}}{\overline{p}_t p_f} \right)^{\sigma_A} \right\} + \sum_g \sum_h cma_{T,g,h} \overline{p}_{T_{last}} (1-\theta_c) \overline{cma}_{T,g,h}. \tag{A17}$$

Market-clearing condition of the total consumption:

$$y_{z,t,g,h} = y_{u,g,h} \left( \frac{p_{u,g,h} \overline{p}_{z,t,g,h}}{p_{z,t,g,h}} \right)^{\frac{1}{\theta}}. \tag{A18}$$

Market-clearing condition of intertemporal utility of the composite consumption:

$$y_{u,g,h} \overline{PV}_{z,g,h} = \frac{inc_{g,h}}{p_{u,g,h}}. \tag{A19}$$

***Income balance equations***

Household income of a given generation:

$$inc_{g,h} = \sum_{t=g}^{g+N} \left[ p_{el,t,g,h} \overline{\pi}_{t,g,h} \omega_{t,g,h} + pens_{t,g,h} + p_f \zeta_{t,g,h} \right] + \theta_c \frac{p_{k,T_{first}} \overline{cma}_{0,g,h}}{1+\overline{r}_p} + \\ + (1-\theta_c) p_f \overline{cma}_{0,g,h} + \sum_{a=0}^{a_T} p_{z_T,a,g,h} \overline{z}_{T,a,g,h} z_{T,a,g,h} + \\ + cma_{T,g,h} p_f \left( -\overline{p}_{T_{last}} (1-\theta_c) \overline{cma}_{T,g,h} \right) + k_T p_{k,T} \left( -\theta_c \overline{cma}_{T,g,h} \right) - \\ - \sum_{t=g}^{g+N} p_{el,t,g,h} liqcons_{g,h} + \sum_{t=g}^{g+N} p_{el,t,g,h} \overline{\pi}_{t,g,h} liqcons_{g,h}. \tag{A20}$$

Government income:

$$
\begin{aligned}
inc_{G,t} = {} & \overline{p}_t \overline{q}_t \left(\overline{D}_0 - \overline{TR}_0\right) p_f + \sum_h \sum_g \left(t_l p_{el,t,g,h} y_{l,t,g,h}\right) + \\
& + t_r p_{r,t} \sum_s \left[ \overline{R}_{0,s} y_{t,s} \left( \frac{p_{va,t,s}(1+\overline{t}_r)}{p_{r,t}\left(1+t_r+\tau_{r,t}\right)} \right)^{\sigma_{kl}} \right] + \\
& + \sum_s \left\{ \frac{p_{A,t,s} t_{VAT,s}}{\left(1+\overline{t}_{VAT,s}\right)} \sum_h \sum_g \left[ \beta_{h,s} \overline{c}_{t,g,h} y_{c,t,g,h} \left( \frac{p_{c,t,g,h}\left(1+\overline{t}_{VAT,s}\right)}{p_{A,t,s}\left(1+t_{VAT,s}+\tau_{VAT,t}\right)} \right)^{\sigma_{cc}} \right] \right\} + \\
& + \sum_s \left( t_{VAT,s} p_{A,t,s} \overline{g}_{0,s} y_{G,t} p_{G,t} \frac{1+\overline{t}_{VAT,s}}{p_{A,t,s}\left(1+t_{VAT,s}+\tau_{VAT,t}\right)} \right) + \\
& + \sum_g \sum_h \left[ \overline{c}_{t,g,h} y_{z,t,g,h} \left( \frac{p_{z,t,g,h} \overline{p}_t}{p_{c,t,g,h}\left(1+\tau_{c,t}\right) \overline{p}_{z,t,g,h}} \right)^{\sigma_{cl}} p_{c,t,g,h} \tau_{c,t} \right].
\end{aligned}
\tag{A21}
$$

***Model restrictions***

Terminal condition of intertemporal utility of the composite consumption:

$$y_{u,g,h} - y_{u,g-1,h} = 0, \quad \forall g > T_{last} - N. \tag{A22}$$

Terminal condition of the total consumption:

$$p_{z,T_{last},g-\mathrm{ord}[a]-1,h} = p_{z_T,a,g,h} \left(1+\overline{r}\right)^{\mathrm{ord}[a]-1}. \tag{A23}$$

Terminal condition of the investment:

$$y_{inv,T_{last}} = \left(1+\gamma\right) y_{inv,T_{last}-1}. \tag{A24}$$

Steady-state growth of the government expenditure:

$$p_{A,t} \overline{q}_t \overline{g}_0 = inc_{G,t}. \tag{A25}$$

## Appendix 2: Overview of Variables of the Model SIOLG 2.0

### *Indices of variables*

$t$ index of the time period (1, …, $T$)
$g$ index of the generation (1, …, $G$)
$h$ index of the household (1, …, $H$)
$s$ index of the production sector (1, …, $S$)
$N$ length of the lifespan of generations

### *Elasticities and other parameters*

$\sigma$ elasticity of intertemporal substitution
$\sigma_{cl}$ elasticity of substitution between consumption and leisure
$\sigma_{cc}$ elasticity of substitution between the components of consumption
$\sigma_{kl}$ elasticity of substitution between capital and labour
$\sigma_A$ Armington's elasticity of substitution
$\eta_X$ elasticity of transformation between exports and domestic production
$\theta$ parameter of the relative risk aversion
$\gamma$ steady state growth rate
$\delta$ depreciation rate
$\upsilon$ utility discount rate
$\bar{r}$ (reference) annual interest rate
$\bar{r}_p$ (reference) interest rate of the periodic interval
$\alpha_h$ consumption preference parameter
$\beta_{h,s}$ share of the good of a given sector in the material consumption
$\theta_c$ share of capital stock in total assets

### *Prices*

$\bar{p}_t$ reference steady state price
$\bar{p}_{T_{last}}$ reference price in the last model period
$p_f$ price of the foreign currency
$\bar{p}_f$ reference price of the foreign currency
$p_{c,t,g,h}$ price of the private consumption
$\bar{p}_{c,t,g,h}$ reference price of the private consumption
$p_{z,t,g,h}$ price of the total consumption
$\bar{p}_{z,t,g,h}$ reference price of the total consumption
$p_{z_T,a,g,h}$ terminal price of the total consumption
$p_{G,t}$ price of the government consumption
$p_{u,g,h}$ price of the intertemporal utility of the composite consumption
$\bar{p}_{u,g,h}$ reference price of the intertemporal utility of the composite consumption
$p_{el,t,g,h}$ price of leisure
$\bar{p}_{el,t,g,h}$ reference price of leisure
$p_{l,t}$ gross price of labour

$\overline{p}_{l,t}$ reference gross price of labour
$p_{r,t}$ price of the capital services
$\overline{p}_{r,t}$ reference price of the capital services
$p_{y,t}$ price of the domestic production
$\overline{p}_{y,t}$ reference price of the domestic production
$p_{A,ts}$ price of the Armington good
$\overline{p}_{A,t,s}$ reference price of the Armington good
$p_{H,t}$ price of the production for domestic market
$\overline{p}_{H,t}$ reference price of the production for domestic market
$p_{va,t,s}$ price of the value added
$\overline{p}_{va,t,s}$ reference price of the value added
$p_{k,t}$ price of the capital stock
$p_{k,T}$ terminal price of the capital stock
$p_{inv,t}$ price of the investment
$p_{M,t}$ price of imports
$\overline{p}_{M,t}$ reference price of imports

### *Activity levels*

$\overline{q}_t$ reference steady state level
$y_{c,t,g,h}$ level of the private consumption
$\overline{y}_{c,t,g,h}$ reference level of the private consumption
$y_{z,t,g,h}$ level of the total consumption
$\overline{y}_{z,t,g,h}$ reference level of the total consumption
$y_{G,t}$ level of the government consumption
$y_{u,g,h}$ level of the intertemporal utility of the composite consumption
$\overline{y}_{u,g,h}$ reference level of the intertemporal utility of the composite consumption
$y_t$ level of the domestic production
$\overline{y}_t$ reference level of the domestic production
$y_{l,t}$ level of the labour supply
$\overline{y}_{l,t}$ reference level of the labour supply
$y_{A,t}$ level of the Armington good
$\overline{y}_{A,t}$ reference level of the Armington good
$y_{k,t}$ level of the capital stock
$y_{inv,t}$ level of the investment
$y_{inv,Tlast}$ level of the investment in the last model period

### *Consumption*

$c_{t,g,h,s}$ private consumption
$\overline{c}_{t,g,h,s}$ reference private consumption
$z_{t,g,h}$ total consumption
$\overline{z}_{t,g,h}$ reference total consumption
$z_{T,a,g,h}$ terminal total consumption
$\overline{z}_{T,a,g,h}$ reference terminal total consumption

$g_t$ government consumption
$\overline{g}_t$ reference government consumption
$u_{g,h}$ intertemporal utility of the composite consumption
$\overline{PV}_{z,g,h}$ reference present value of the total consumption

***Production and foreign trade***

$\omega_{t,g,h}$ labour endowment
$\overline{\pi}_{t,g,h}$ reference labour productivity profile
$l_{t,g,h}$ labour profile
$\overline{l}_{t,g,h}$ reference labour profile
$ls_{t,g,h}$ leisure profile
$\overline{ls}_{t,g,h}$ reference leisure profile
$\overline{w}_t$ average net wage
$N_{w,g}$ average number of years of service of a given generation
$id_{ag,t,s}$ aggregate intermediate demand
$\overline{id}_{ag,t,s}$ reference aggregate intermediate demand
$va_t$ value added
$\overline{va}_t$ reference value added
$k_t$ capital stock
$\overline{k}_t$ reference capital stock
$k_T$ terminal capital stock
$\overline{k}_T$ reference terminal capital stock
$R_{t,s}$ capital services
$\overline{R}_{t,s}$ reference capital services
$inv_t$ investment
$\overline{inv}_t$ reference investment
$H_t$ production for the domestic market
$\overline{H}_t$ reference production for the domestic market
$X_t$ exports
$\overline{X}_t$ reference exports
$M_t$ imports
$\overline{M}_t$ reference imports

***Income and wealth***

$inc_{g,h}$ income of a household of a given generation
$inc_{IPDI,t}$ income of the Institute of Pension and Disability Insurance
$inc_{G,t}$ income of the government
$\overline{D}_t$ reference government budget deficit
$cma_{t,g,h}$ reference asset position
$cma_{T,g,h}$ terminal asset position
$\overline{cma}_{T,g,h}$ reference terminal asset position
$CMA$ total assets

### *Taxes, contributions and transfers*

| | |
|---|---|
| $t_l$ | tax rate of the labour income tax |
| $t_l^{rent}$ | contribution rate of the mandatory pension contributions |
| $\tau_{l,t}^{rent}$ | replacement contribution rate of the mandatory pension contributions |
| $t_r$ | tax rate of the capital income tax |
| $\bar{t}_r$ | reference tax rate of the capital income tax |
| $\tau_{r,t}$ | replacement tax rate of the capital income tax |
| $t_{VAT}$ | tax rate of the value-added tax |
| $\bar{t}_{VAT}$ | reference tax rate of the value-added tax |
| $\tau_{VAT,t}$ | replacement tax rate of the value-added tax |
| $\tau_{c,t}$ | tax rate of the consumption tax for adjustment of government expenditure |
| $\overline{TR}_t$ | reference transfers of the government |
| $\xi_{t,g,h}$ | transfer of the government to a household of a given generation |
| $\bar{\xi}_{t,g,h}$ | reference transfer of the government to a household of a given generation |
| $\bar{\xi}_{IPDI,t}$ | reference transfer of the government to the Institute of Pension and Disability Insurance |

### *The pension system*

| | |
|---|---|
| $pens_{t,g,h}$ | pension of a household of a given generation |
| $\overline{pens}_{t,g,h}$ | reference pension of a household of a given generation |
| $aggpens_{t,g,h}$ | aggregate pension, paid to a household of a given generation |
| $v_{g,h}^{rh}$ | number of retired households |
| $\theta_{g,h}^{\#}$ | number of adults in a household |
| $\alpha_g$ | accrual rate for calculating the pension of a given generation |
| $\tilde{\alpha}_g$ | target accrual rate for calculating the pension of a given generation |
| $vtax_t$ | valuation tax |
| $K_{Wl}$ | coefficient of adjustment of the growth of pensions with respect to the growth of wages |
| $\vartheta^{min}$ | net wage factor for the calculation of the minimum pension base |
| $\xi$ | correction factor for the calculation of the pension |
| $\wp$ | pension base divisor, including the calibration correction |
| $\overline{pb}_{g,h}$ | pension base for calculating the pension of a given generation |
| $pb_{g,h}^{mm}$ | pension base for calculating the pension of a given generation with a correction for the minimum and maximum pension base |
| $pb^{min}$ | minimum pension base in the base time period |
| $pb_g^{min}$ | minimum pension base of a given generation |
| $pb_t^{min}$ | minimum pension base |
| $pb_t^{max}$ | maximum pension base |
| $\iota_{t,g}^{rent}$ | pension index for calculation of the pension of a given generation |
| $\iota_t^{min}$ | pension index for calculation of the minimum pension base |
| $\tilde{\iota}_t$ | generation-independent pension indexation factor |
| $\Im_g$ | ratio of interest-bearing to non-interest-bearing supplementary pension insurance savings |

$liqcons_{g,h}$ share of the net wage, assigned for savings in the second pension pillar (supplementary pension insurance)

$\overline{spilar}_g$ pension from supplementary pension insurance as a proportion of the pension from mandatory pension insurance in the reference scenario

***Multipliers***

$\mu_B$ multiplier of the benchmark scenario

$\mu^{sp}$ multiplier of the counterfactual scenario with active second pension pillar

$\mu_{t,g}^{aret}$ multiplier for the generation that is already retired in a given year

$\mu_{t,g}^{ret}$ multiplier for the generation that is retiring in a given year

$\mu_{t,g,h}^{alive}$ multiplier for the household that already exists in the first model period

$\mu_{t,g,h}^{wh}$ multiplier for the household that lives entirely within the model horizon