On the Role of Memory in Asset Pricing Models with Heterogeneous Beliefs

There is an important paradigm shift taking place in economics and finance in the last decade; from the representative, rational agent approach towards a behavioural, agent-based approach with heterogeneous boundedly-rational agents. While the traditional approach makes use of simple, analytically tractable models with a representative, perfectly rational agent, the new behavioural approach utilizes computational and numerical methods on agent-based simulation models. By now, there is a rather extensive literature available on computationally oriented agent-based simulation models of artificial markets (*cf*. LeBaron, 2006). However, there is also an important stream developing in the literature, which endeavours to maintain at least to same extent analytically tractable heterogeneous agent models, for which theoretical results are obtained supporting numerical simulation results (*cf*. Hommes, 2006). Such an approach uses a mixture of analytic and computational tools of nonlinear economic dynamics.

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Heterogeneous agent models are present in various fields of economic analysis, such as market maker models, exchange rate models, monetary policy models, overlapping generations models, and experimental models of socio-economic behaviour. Yet the field with the most systematic and perhaps most promising nonlinear dynamic approach seems to be asset price modelling. Contributions of Brock and Hommes (1998), LeBaron (2000), Brock and Hommes (2001), Hommes *et al*. (2002), Chiarella and He (2002), Gaunersdorfer *et al*. (2003), and Brock *et al*. (2005) thoroughly demonstrate how a simple standard pricing model is able to lead to complex dynamics that make it extremely hard to predict the evolution of prices in asset markets. The main framework of analysis of such asset pricing models constitutes a financial market application of the evolutionary selection of expectation rules, introduced by Brock and Hommes (1997) and called the adaptive belief system.

As a model in which different agents have the ability to switch beliefs, the adaptive belief system in a standard discounted value asset pricing set-up is derived from mean-variance maximization and extended to the case of heterogeneous beliefs (*cf*. Brock and Hommes, 1998). It can be formulated in terms of deviations from a benchmark fundamental and therefore used in experimental and empirical testing of deviations from the rational expectations benchmark. Agents are boundedly rational, act independently of each other and select a forecasting or investment strategy based upon its recent relative performance. The key feature of such systems, which often incorporate active learning and adaptation, is endogenous heterogeneity (LeBaron, 2002), which means that markets can move through periods that support diverse population of beliefs, and others where these beliefs and strategies might collapse down to a very small set.

While recent literature on asset price modelling focuses mainly on impacts of heterogeneity of beliefs in the standard adaptive belief system as set up by Brock and Hommes (1997) on the market dynamics and stability on one hand, and the possibility of survival of such 'irrational', speculative traders in the market on the other, several crucial issues regarding the foundations of asset price modelling and its underlying theoretical findings remain open and indeterminate. We believe one of such issues is related to heterogeneity in investors' time horizon; both their planning and their evaluation perspective.

Namely, it has been scarcely addressed so far how memory in the fitness measure and in expectation rules, i.e. the share of past information that boundedly rational economic agents take into account as decisions makers, affects stability of evolutionary adaptive systems and survival of technical trading. Our motivation was therefore to lay foundations for a competent and critical theoretical analysis of setting this modelling assumption in a simple, analytically tractable asset pricing model, similar to the one of Brock and Hommes (1998).

Asset Pricing Models and Heterogeneous Beliefs

The adaptive belief system employs a mechanism, which deals with interaction between fractions of market traders of different types, and distance between the fundamental and the actual price. Financial markets are thus viewed as an evolutionary system, where price fluctuations are driven by an evolutionary dynamics between different expectation schemes. The model can be viewed as composed of two simultaneous parts; present value asset pricing, which results in an equilibrium pricing equation, and evolutionary selection of strategies, which results in a fractions of belief types equation.

What determines different types of heterogeneous agents in an adaptive belief system is the particular form of deterministic function in the forecasting (expectation) rule. In general, we distinguish between two typical investor types; fundamentalists and 'noise traders' or 'chartists' or technical analysts. Fundamentalists believe that the price of an asset is defined solely by its efficiency market hypothesis fundamental value, i.e. the present value of the stream of future dividends. Since by definition they have no knowledge about other beliefs and fractions, their deterministic function equals zero. Technical analysts, on the other hand, believe that asset prices are not completely determined by the fundamentals, but may be predicted by inferences on past prices. Depending on the purpose of the analysis, it is possible to distinguish between (pure) trend chasers, (pure) contrarians, (pure) biased beliefs, and combinations thereof. Since the deterministic function in the expectation rule depends on preceding price deviations, it can also be seen as including memory. However, due to rapidly increasing analytical complexity, which will be put out in due course, this issue has so far mainly been neglected.

Memory and Performance of Heterogeneous Agents

In an asset pricing model the fitness function or performance measure of each trader type can be defined in terms of its realized profits. In fact, it can be expressed as the weighted sum of realized profits, i.e. as the sum of current realized profits and a share of past fitness, which is in turn defined as past realized profits. Albeit introducing additional analytical complexity, we usually also take into account the costs for predictor of particular trader type, since more information-intense predictors are evidently more costly. The share of past fitness in the performance measure is expressed by the parameter called memory strength. When the value of this parameter is taken to be zero, the fitness is given by most recent net realized profit. Due to analytical tractability this is presently for the most part the case in the existing literature on asset pricing models with heterogeneous agents. When the memory strength parameter takes a positive value, certain share of current realized profits in any given period is taken into account when calculating the performance measure in the next time period. In case that the value of memory strength parameter amounts to one, the entire accumulated wealth is taken into account, which may mean that a particular group of traders has rational expectations (perfect foresight).

> *"(The lack of) memory is an important aspect of the market that keeps it from converging and prevents the elimination of speculative strategies"*

The core of the model in deviations from the fundamental price thus consists of three main sections; the equilibrium pricing equation with incorporated heterogeneous beliefs, the fractions of belief types equation, and the fitness measure equation. Memory strength indeed appears in the latter, but memory in fact spreads through the model using appropriate deterministic functions representing different belief types. Since it primarily represents the share of past fitness in the performance measure, memory can also be thought of as a share of past information that boundedly rational economic agents take into account as decision makers. When the value of the memory strength parameter is nonzero, past realized profits take an active role in determining the asset price in the simultaneous adaptive belief system. Namely, with memory affected fitness measure of each trader type enters discrete choice probability equation, which determines fractions of respective belief types, consequently affecting the asset price, which is modelled as the sum over trader types of products of a fraction of particular trader type and its deterministic function. Memory strength is thus one of the parameters of the asset pricing model that decisively influence our inferences on both stability of evolutionary adaptive systems and survival of technical trading.

Short-memory Trading and Market Volatility

Following LeBaron (2002), we can reproduce and analyze some of the scarce findings that were established to date on changing memory in heterogeneous agent models. In the original approach simulated agent-based financial markets were used of individuals following relatively simple behavioural rules that are updated over time. Actually, time is an essential and critical feature of any such model. It can be argued that someone believing that the world is stationary should use all available information in forming his or her beliefs, while if one views the world as constantly in a state of change, then it will be better to use time series reaching a shorter length into the past (cf. LeBaron, 2002). The dilemma is thus seen as an evolutionary challenge where long-memory agents, using lots of past data, are pitted against short-memory agents to see who takes over the market.

The market simulation to be used here was consisting of traders with many different memory lengths, operationalized as a proportion analogue of drawings from a uniform distribution between six months and twenty years. Four variables were being examined; logarithms of the price series, trading volume in units of shares subjected to trade, returns, and dividend-price ratio. The price series indeed exhibited the expected linear trend driven by the constant dividend growth, but the prices seemed to take large deviations around this trend. Furthermore, volume was not a large fraction of the shares outstanding, but it was not going to zero as it should if the agents' beliefs had been converging to each other. Returns also demonstrated some features of actual markets, since there were large spikes corresponding to large up and down movements in the market. These movements corresponded to the well documented nongausseineity of financial return series. Also, the volatility in the market seemed to be clumped with periods of relative calm and periods of large activity. The dividend-price ratio, which compared movements of the equity price series with its underlying fundamental, should have been a constant if there were no changes in the underlying riskiness of the equity security. Yet from the simulation results it was clear that large and persistent deviations had occurred.

Long-memory Trading and Market Stability

The second market simulation was performed in the same set-up of financial markets, composed of individuals with heterogeneous behavioural rules, as proposed by LeBaron (2002). However, the agents' memory lengths were now restricted to fit the proportion analogue of drawings from a uniform distribution between 16 and 20 years. With the use of this simulation, where the population of agents was long memory, it has been examined whether much of the variability and instability in the market was really coming from the presence of short-memory traders. Again, the same four variables were being examined. The conjecture was generally being confirmed. Namely, the price series was much more stable, while the trading volume was near zero, except for a few brief jumps. Moreover, the returns were also generally stable with the exception of a few jumps, and the dividend-price ratio was very close to constant. Numerical results can be checked by theoretical calculations, where one is typically able to establish that the market is approaching the theoretical benchmark of the well-defined homogeneous agent equilibrium.

As already mentioned, memory in the expectation rules is somewhat less analytically tractable phenomenon, since including more preceding price deviations immediately increases the dimension of the system. This is indeed also the case with memory in the fitness measure, but there the performance measure can be written as weighted sum of contemporaneous realized profits and past fitness, thus keeping the increase in dimension under control. Usually only one memory lag is taken into account in the expectation rule to attain analytical tractability, but an appropriate numerical analysis can always be employed. Currently, we can only speculate about the effects of changing memory in expectation rules on stability of evolutionary adaptive systems and survival of technical trading; especially in interaction with changing memory in the fitness measure. We could expect that incorporating much memory in the expectation rule with more or less equal weights given to past prices produces an average price forecast close to the fundamental value. On the

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other hand, with most weight given to the last observations, the expectation rule is likely to be more of the trend following kind with all the accompanying consequences.

Concluding Remarks

Memory strength, which is of our primary concern in this article, represents the share of past fitness in the performance measure of an asset pricing model, which enters as an element the discrete choice probability equation. The latter then determines fractions of respective belief types, consequently affecting the asset price. On the other hand, memory is also widespread in the model in the form of past price deviations taken into account in expectation rules. Memory strength is thus one of the parameters of asset pricing models that decisively influence our inferences on both stability of evolutionary adaptive systems and survival of technical trading. The cause of much of the above described dynamics can be related to the interaction between traders with differing views of the past. Agents with a short-term perspective can both influence the market in terms of increasing volatility and create an evolutionary space where they are able to prosper. Changing the population to more long-memory types leads to a reliable convergence in strategies, which is a useful benchmark test. Memory is therefore an important aspect of the market that keeps it from converging and prevents the elimination of speculative strategies.

We can observe that computational models are becoming increasingly important, since they allow many aspects at the micro level and details of the interaction among agents to be modelled and simulated. Heterogeneity is likely to play a key role in this approach, and agent-based computational asset pricing models thus deserve high priority in future work. But a problem with such simulation models is that there tend to be too many degrees of freedom and too many parameters. One of such issues that we shall pursue to make less indeterminate and unresolved therefore relates to memory strength. When the consequences of changing memory on stability of evolutionary adaptive systems and survival of technical trading are better understood, we will be able to formulate asset pricing models in a more consistent and efficient manner. Optimistically, expanded and improved these models may yield further (seemingly) counter-intuitive results in terms of methods to stabilize, predict, or improve on current market institutions.

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